DESIGN FOR DIGITAL COMMUNICATION SYSTEMS VIA SAMPLED-DATA H^∞ CONTROL

M. Nagahara [*,1] **Y. Yamamoto** [*,2]

[*] *Department of Applied Analysis and Complex Dynamical Systems, Graduate School of Informatics, Kyoto University, Kyoto 606-8501, JAPAN*

Abstract: The design procedure for the equalization of digital communication channels is developed based on the sampled-data H^∞ control theory. The procedure provides transmitting/receiving filters so as to minimize the error between the original signal and the received signal with a time delay, and to reduce the noise added to the channel. While the system has an ideal sampler, a zero-order hold and a time delay, the design problem can be reduced to a finite-dimensional discrete-time problem using the FSFH (fast-sample and fast-hold) approximation. Numerical examples are presented to illustrate the effectiveness of the proposed method.

Keywords: Sampled-data systems; Digital communications; Digital filters; H-infinity optimization; Time delay

1. INTRODUCTION

Nowadays the importance of digital communications is increasing owing to the rapid growth of the Internet, the cellular phones, and so on(Proakis, 1989). In digital communication, especially in pulse amplitude modulation (PAM) or in pulse code modulation (PCM), the analog signal which is to be transmitted is sampled and becomes a discrete-time signal. In the conventional way, the characteristic of the analog signal is not considered, and hence the whole system is regarded as a discrete-time system. For example, one usually assumes that the original analog signal is band-limited up to the Nyquist frequency. But in reality no signals are entirely band-limited.

This paper proposes a new design methodology based on the sampled-data control theory(Chen and Francis, 1995*b*) that takes account of inter-sample behaviors or frequency components beyond the Nyquist frequency in discrete-time. Recently, the sampled-data control theory is applied to some digital signal processing systems(Chen and Francis, 1995*a*; Khargonekar and Yamamoto, 1996; Yamamoto *et al.*, 1997; Nagahara and Yamamoto, 2000). We propose a design for the digital communication system via the sampled-data control. In (Erdogan *et al.*, 2000) a discrete-time H^∞ design for receiving filters or equalizers is introduced, but no design for transmitting filter is mentioned. However, it is difficult to attenuate both the signal reconstruction error and the additive noise by only an equalizing after the signal is received. Therefore we show the transmitting filter design which is executed in the same way as the receiving filter design. Moreover, we introduce the H^∞ method which takes account of a tradeoff between the signal reconstruction error and the energy of transmitted signal with an appropriate weighting function. Design examples are presented to illustrate the effectiveness of the proposed method.

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[1] nagahara@acs.i.kyoto-u.ac.jp

[2] yy@i.kyoto-u.ac.jp; author to whom all correspondence should be addressed.

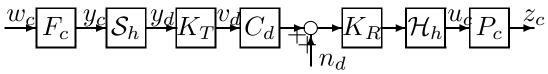

Fig. 1. Digital communication system.

2. DESIGN PROBLEM FORMULATION

The block diagram Figure 1 shows a digital communication system which is known as PAM or PCM. The incoming signal $w_c \in L^2[0, \infty)$ goes through an analog low-pass filter F_c and becomes y_c which is nearly (but not entirely) band limited. The filter F_c governs the frequency-domain characteristic of the analog signal y_c [3]. The signal y_c is then sampled by the sampler $\mathcal{S}_h$ to become a discrete-time signal (or PAM signal) y_d with sampling period h. Then the signal is shaped or enhanced by the transmitting digital filter K_T to the signal v_d to be transmitted to a communication channel.

The transmitted signal v_d is corrupted by the communication channel C_d and the additive noise n_d. In PCM communication, n_d is also considered as the noise generated by quantizing and coding error. The received signal goes through the receiving digital filter K_R which tries to attenuate the corruption and the noise, then becomes an analog signal u_c by the hold device $\mathcal{H}_h$ with sampling period h and smoothed by an analog low-pass filter P_c and finally we have the output signal z_c.

Our objective is to reconstruct the original analog signal y_c by the transmitting filter K_T and the receiving filter K_R against the corruption caused by the channel C_d and the additive noise n_d. Therefore consider the block diagram Figure 2 which is the signal reconstruction error system for the design. In the diagram the following points are taken into account:

- The time delay e^{-Ls} is introduced because we allow a certain amount of time delay for signal reconstruction.
- The transmitted signal v_d is estimated with a weighting function W_z because the energy or the amplitude of the transmitted signal v_d is usually limited.
- The noise has a frequency characteristic W_n.

Then our design problem is as follows:

Problem 1. Given stable analog filters $F(s)$ and $P(s)$, digital filters (weighting functions) $W_n(z)$ and $W_z(z)$ and a channel model $C_d(z)$, find digital filters $K_T(z)$ and $K_R(z)$ which minimizes

$$J^2 := \sup_{w_c \in L^2, n_d \in l^2} \frac{\|e_c\|_{L^2}^2 + \|z_d\|_{l^2}^2}{\|w_c\|_{L^2}^2 + \|n_d\|_{l^2}^2}. \qquad (1)$$

[3] In the conventional design F_c is considered as an ideal filter which has a cut-off frequency up to the Nyquist frequency.

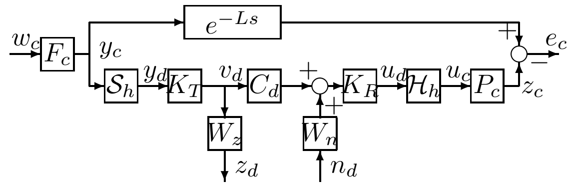

Fig. 2. Signal reconstruction error system.

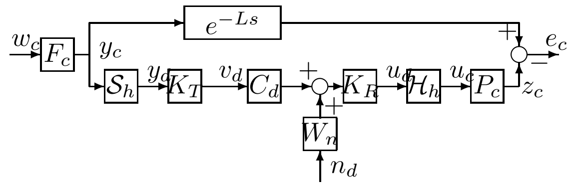

Fig. 3. Error System $\mathcal{T}_R$ for receiving filter design

3. DESIGN ALGORITHM

3.1 *Decomposing Design Problems*

Problem 1 is a simultaneous design problem of a transmitting filter and a receiving filter, and it is difficult to solve the problem directly. Therefore we introduce a decomposition of the design problem into two steps, that is the design for the receiving filter and that for the transmitting filter.

Obviously the transmitting filter K_T cannot attenuate the additive noise n_d, hence the receiving filter K_R has to play that role. Moreover K_R has to reconstruct the original signal from the corrupted signal (if K_R did not have to reconstruct, the optimal filter will be clearly $K_R = 0$) Therefore we first design the receiving filter K_R in order to reconstruct the original signal and to attenuate the noise by the block diagram Figure 2 with $W_z = 0$ and with $K_T = 1$. Then design the transmitting filter by the block diagram Figure 2 with $W_n = 0$ and with K_R which is obtained the previous design, that is we consider the channel as $K_R C_d$.

The design procedure is as follows:

Step 1(Design for receiving filter) Find a receiving filter K_R which minimizes

$$\|\mathcal{T}_R\|_\infty^2 := \sup_{w_c \in L^2, n_d \in l^2} \frac{\|e_c\|_{L^2}^2}{\|w_c\|_{L^2}^2 + \|n_d\|_{l^2}^2}, \qquad (2)$$

in Figure 3 with fixed K_T (the initial filter is $K_T = 1$).

Step 2(Design for transmitting filter) Find a transmitting filter K_T which minimizes

$$\|\mathcal{T}_T\|_\infty^2 := \sup_{w_c \in L^2} \frac{\|e_c\|_{L^2}^2 + \|z_d\|_{l^2}^2}{\|w_c\|_{L^2}^2}, \qquad (3)$$

in Figure 4 with K_R which is obtained in the previous step.

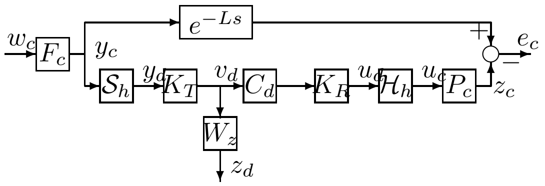

Fig. 4. Error System $\mathcal{T}_T$ for transmitting filter design

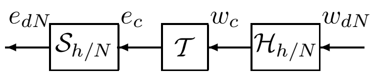

Fig. 5. fast sample/hold discretization

3.2 *Fast Sample/Hold Approximation*

The design problems (2) and (3) involve a continuous time delay component e^{-Ls}, and hence they are infinite-dimensional sampled-data problems. To avoid this difficulty, we employ the fast sample/hold approximation method (Keller and Anderson, 1992; Yamamoto *et al.*, 1999). By the method, our design problems (2) and (3) are approximated to finite-dimensional discrete-time problems assuming that the delay time L to be mh where m is a positive integer:

Theorem 2. Assume that $L = mh$, $m \in \mathbb{N}$. Then,

(1) for the error system $\mathcal{T}_R$ in Step 1, there exist finite-dimensional discrete-time systems $\{T_{R,N} : N = 1, 2, \dots\}$ such that

$$\lim_{N\to\infty} \|T_{R,N}\|_\infty = \|\mathcal{T}_R\|_\infty.$$

(2) for the error system $\mathcal{T}_T$ in Step 2, there exist finite-dimensional discrete-time systems $\{T_{T,N} : N = 1, 2, \dots\}$ such that

$$\lim_{N\to\infty} \|T_{T,N}\|_\infty = \|\mathcal{T}_T\|_\infty.$$

PROOF. By the fast sample/hold method, we approximate continuous-time inputs and outputs to discrete-time ones via the ideal sampler and the zero-order hold that operate in the period h/N (Figure 5). Then apply the discrete-time lifting(Yamamoto *et al.*, 1997) $\mathbf{L}_N$ to the discretized input/output signal e_{dN} and w_{dN}, we can get the lifted signals

$$\widetilde{e}_{dN} := \mathbf{L}_N(e_{dN}), \quad \widetilde{w}_{dN} := \mathbf{L}_N(w_{dN}).$$

Then we can approximate the continuous signal as

$$\|e_c\|_{L^2} \approx \sqrt{\frac{h}{N}}\|\widetilde{e}_{dN}\|_{l^2}, \quad \|w_c\|_{L^2} \approx \sqrt{\frac{h}{N}}\|\widetilde{w}_{dN}\|_{l^2}.$$

Moreover define

$$\|T_{R,N}\|_\infty^2 := \sup_{\widetilde{w}_{dN}, n_d \in l^2} \frac{\|\widetilde{e}_{dN}\|_{l^2}^2}{\|\widetilde{w}_{dN}\|_{l^2}^2 + \sqrt{\frac{N}{h}}\|n_d\|_{l^2}^2},$$

$$\|T_{T,N}\|_\infty := \sup_{\widetilde{w}_{dN} \in l^2} \frac{\|\widetilde{e}_{dN}\|_{l^2}^2 + \sqrt{\frac{N}{h}}\|z_d\|_{l^2}^2}{\|\widetilde{w}_{dN}\|_{l^2}},$$

where the systems $T_{R,N}$ and $T_{T,N}$ are approximated to finite-dimensional discrete-time systems, then we can show $\|T_{R,N}\|_\infty \to \|\mathcal{T}_R\|_\infty$, $\|T_{T,N}\|_\infty \to \|\mathcal{T}_T\|_\infty$ as $N \to \infty$ by using the method as shown in (Yamamoto *et al.*, 1999) under the assumption $L = mh$. □

Once the problems have been reduced to discrete-time problems, they can be solved by a control design toolbox such as those given by MATLAB. The resulting discrete-time approximant is given by the following:

Theorem 3. The approximated discrete-time systems $T_{R,N}$ and $T_{T,N}$ are given as follows:

$$T_{R,N} := \mathcal{F}_l\left(G_{R,N},\ K_R\right),$$
$$T_{T,N} := \mathcal{F}_l\left(G_{T,N},\ K_T\right),$$
$$G_{R,N} := \begin{bmatrix} \left[\, z^{-m}F_{dN},\ 0 \,\right], & -P_{dN} \\ \left[\, C_d K_T J F_{dN},\ W_n \,\right], & 0 \end{bmatrix},$$
$$G_{T,N} := \begin{bmatrix} \begin{bmatrix} z^{-m}F_{dN} \\ 0 \end{bmatrix}, & \begin{bmatrix} -P_{dN}K_R C_d \\ W_z \end{bmatrix} \\ F_{dN}, & 0 \end{bmatrix},$$
$$J := [I, 0, \dots, 0],$$
$$F_{dN}(z) :=$$
$$\left[\begin{array}{c|cccc} A_{Fd}^N & A_{Fd}^{N-1}B_{Fd}, & A_{Fd}^{N-2}B_{Fd}, & \dots & B_{Fd} \\ \hline C_F & 0, & 0, & \dots, & 0 \\ C_F A_{Fd} & C_F B_{Fd}, & 0, & \dots, & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ C_F A_{Fd}^{N-1} & C_F A_{Fd}^{N-2}B_{Fd}, & C_{Fd}A_{Fd}^{N-3}B_{Fd}, & \dots, & 0 \end{array}\right],$$
$$P_{dN}(z) := \left[\begin{array}{c|c} A_{Pd}^N & \sum_{k=1}^{N} A_{Pd}^{N-k}B_{Pd} \\ \hline C_P & D_P \\ C_P A_{Pd} & C_P B_{Pd} + D_P \\ \vdots & \vdots \\ C_P A_{Pd}^{N-1} & \sum_{k=2}^{N} C_P A_{Pd}^{N-k}B_{Pd} + D_P \end{array}\right],$$
$$A_{Fd} := e^{A_F \frac{h}{N}}, \quad B_{Fd} := \int_0^{\frac{h}{N}} e^{A_F t}B_F dt,$$
$$A_{Pd} := e^{A_P \frac{h}{N}}, \quad B_{Pd} := \int_0^{\frac{h}{N}} e^{A_P t}B_P dt,$$
$$F(s) =: \left[\begin{array}{c|c} A_F & B_F \\ \hline C_F & 0 \end{array}\right], \qquad P(s) =: \left[\begin{array}{c|c} A_P & B_P \\ \hline C_P & D_P \end{array}\right].$$

where $\mathcal{F}_l(G, K)$ denotes the linear fractional transformation of plant G and filter K.

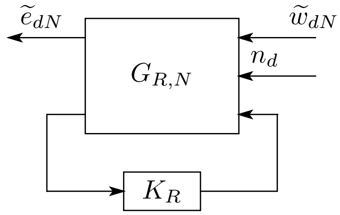

Fig. 6. Discrete-time H^∞ design problem for receiving filter K_R

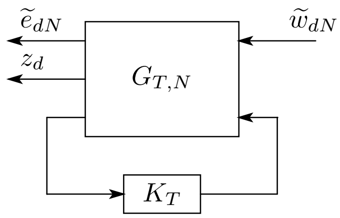

Fig. 7. Discrete-time H^∞ design problem for transmitting filter K_T

Then our design problems (2) and (3) are reduced to finite-dimensional discrete-time H^∞ problems, which are shown in Figure 6 and Figure 7.

4. DESIGN EXAMPLES

4.1 *Design for $W_z = 0$*

We present a design example for

$$F(s) := \frac{1}{10s+1}, \quad P(s) := 1, \quad W_n(z) := 1,$$
$$C_d(z) := 1 + 0.65z^{-1} - 0.52z^{-2} - 0.2975z^{-3},$$

with sampling period $h = 1$ and time delay $L = mh = 2$. An approximate design is executed here for $N = 8$. Here we design without considering the transmitting signal, that is $W_z(z) = 0$. For comparison, the discrete-time H^∞ design(Erdogan *et al.*, 2000) is also done.

Figure 8 shows the gain responses of the filters, and Figure 9 shows the frequency response of T_{ew} which is the system from the input w_c to the error e_c, and Figure 10 shows that of T_{zn} from the additive noise n_d to the output z_c. Compared with the discrete-time design, the sampled-data one shows better frequency response both in $\mathcal{T}_{ew}$ and in $\mathcal{T}_{zn}$. Moreover, we can say that only an equalizer is not able to attenuate the corruption caused by the channel and the additive noise, that is we need an appropriate transmitter for transmission.

To explain this fact, we show a simulation of these communication systems. The input signal y_c is the rectangular wave whose amplitude is 1, and the noise n_d is the discrete-time sinusoid $n_d[k] = \sin(2k)$. Figure 11 shows the output z_c

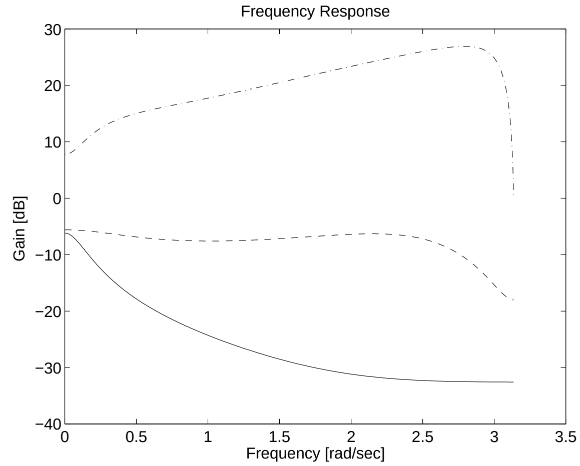

Fig. 8. Gain response of filters:sampled-data H^∞ design (transmitting filter: solid, receiving filter: dots) and discrete-time H^∞ design (dash).

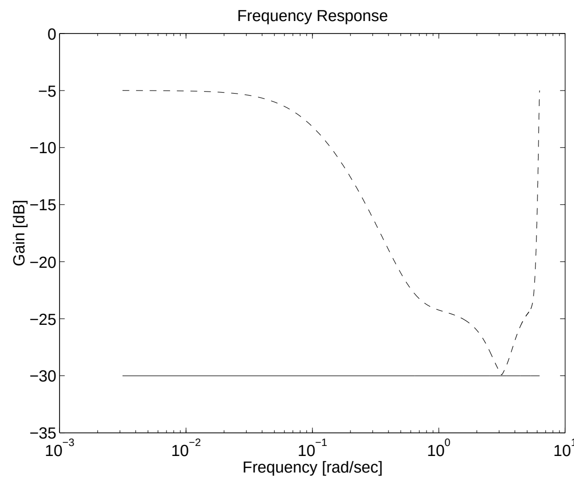

Fig. 9. Frequency response of $\mathcal{T}_{ew}$: sampled-data H^∞ design (solid) and discrete-time H^∞ design (dash).

with the receiving filter and the transmitting filter designed via sampled-data method, and Figure 12 shows that with the receiving filter designed in discrete-time (and without any transmitting filter). We see that the former shows much better reconstruction against the noise than the latter.

4.2 *Design for $W_z(z) \neq 0$*

Then we consider the design with the estimation of the transmitting signal v_d, that is $W_z(z) \neq 0$.

We observe from Figure 8 that the transmitting filter shows high gain around the Nyquist frequency (i.e. $\omega = \pi$), and hence we take

$$W_z(z) = r \cdot \frac{z-1}{z+0.5}$$

as the weighting function of the transmitting signal, whose gain characteristic is shown in Figure

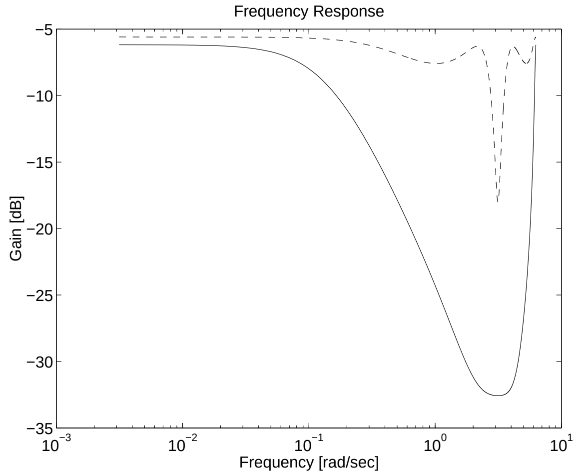

Fig. 10. Frequency response of $\mathcal{T}_{zn}$: sampled-data H^{∞} design (solid) and discrete-time H^{∞} design (dash).

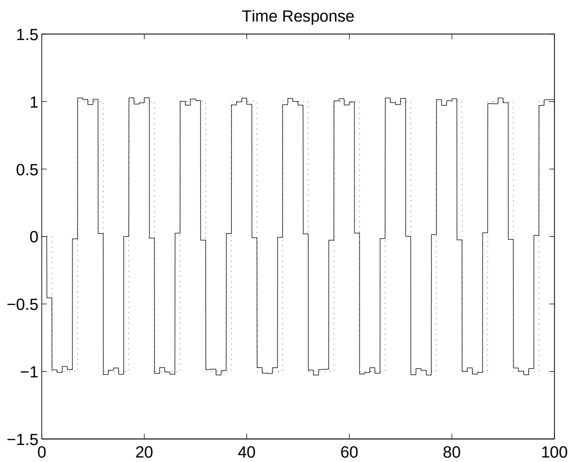

Fig. 11. Time response with sampled-data design.

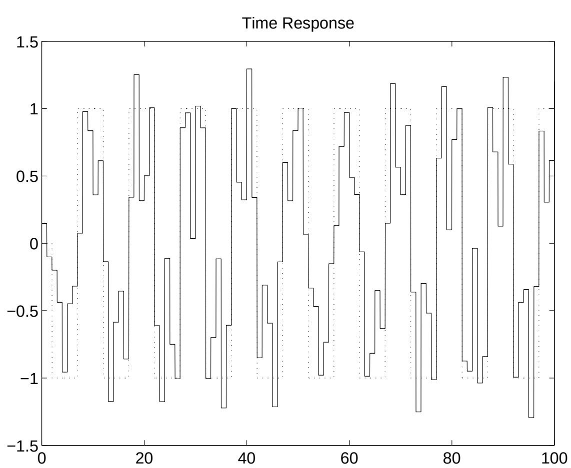

Fig. 12. Time response with discrete-time design.

13 where the parameter $r = 0.21$. The other design parameters are the same as the example above.

Figure 14 shows the H^{∞} norm of $\mathcal{T}_{ew}$ and $\mathcal{T}_{vw}$ which is the system from w_c to v_d in Figure 2 which varies with $r \in [0, 5]$. We can take account

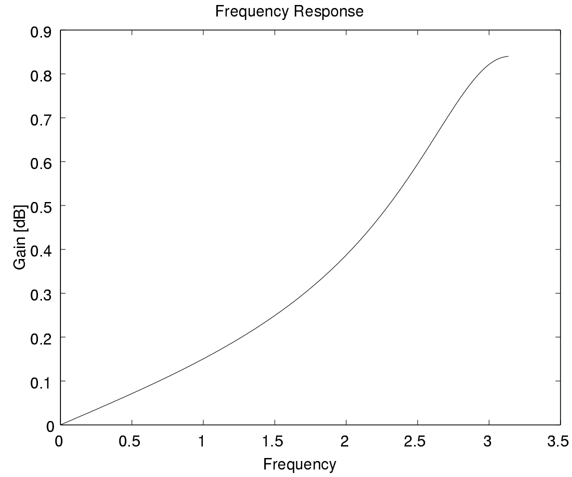

Fig. 13. Gain characteristic of the weighting filter $W_z(z)$.

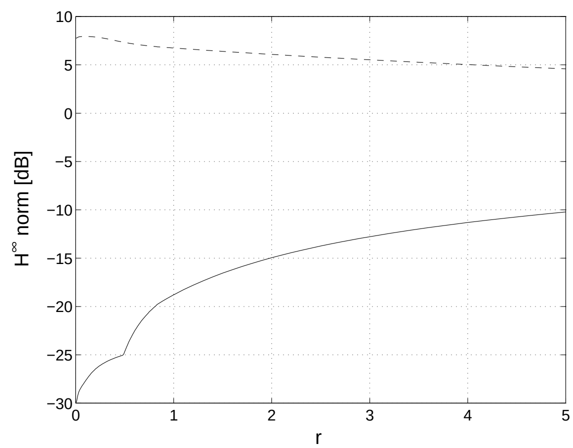

Fig. 14. Relation between r and $\|T_{ew}\|_{\infty}$ (solid), $\|T_{vw}\|_{\infty}$ (dash).

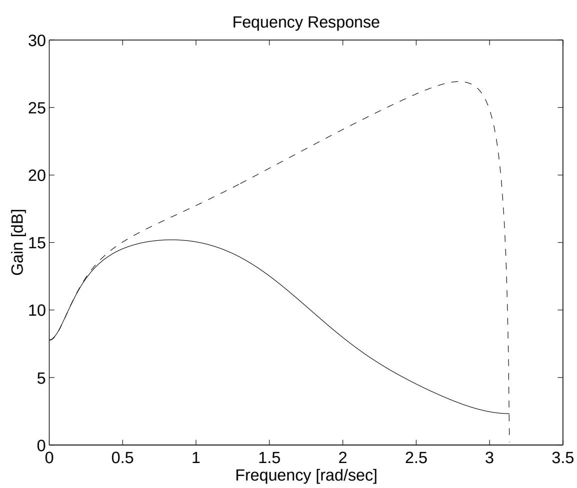

Fig. 15. Gain response of transmitting filters designed for $r = 0.21$ (solid) and $r = 0$ (dash)

of a trade-off between the error attenuation level and the amount of the transmitting signal with Figure 14. For example, we choose $r = 0.21$ in order to attenuate the error less than -26dB.

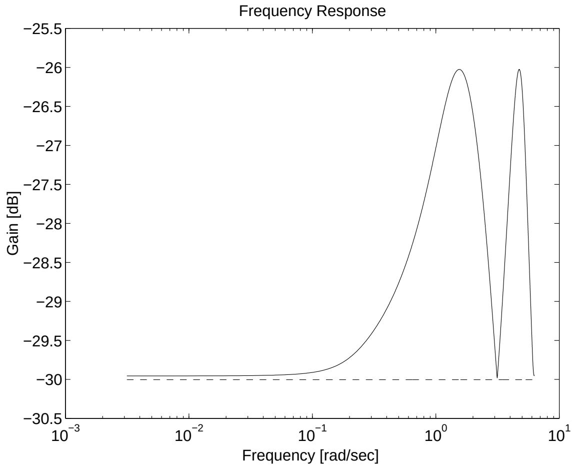

Fig. 16. Frequency response of $\mathcal{T}_{ew}$ designed for $r = 0.21$ (solid) and $r = 0$ (dash).

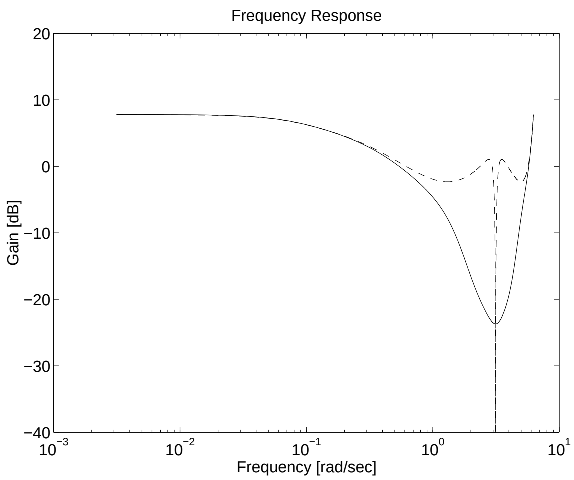

Fig. 17. Frequency response of $\mathcal{T}_{vw}$ designed for $r = 0.21$ (solid) and $r = 0$ (dash)

Figure 15 shows the gain response of transmitting filters designed for $r = 0$ and $r = 0.21$. We can see that the new filter shows better attenuation than the filter designed for $r = 0$ at high frequency.

Figure 16 shows the frequency response of the error system $\mathcal{T}_{ew}$. We see that the attenuation level of $\mathcal{T}_{ew}$ designed for $r = 0.21$ is less than -26dB. Figure 17 shows the frequency response of $\mathcal{T}_{vw}$. We can see that the amount of the transmitting signal is attenuated at high frequency.

5. CONCLUDING REMARKS

We have presented a new method of designing transmitting/receiving filter in digital communication. An advantage here is that an analog otimal performance can be obtained, and this can be advantageous in audio/speech signal transmission. Another advantage is that the trade-off between the attenuation of the reconstruction error and the energy of the transmitting signal is considered by the H^∞ design with an appropriate weighting function. By the fast sample/hold method, the design is reduced to a finite dimensional discrete time design, which can be easily implemented to CAD (e.g. MATLAB).

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