Compressive Sampling for Networked Feedback Control

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Abstract

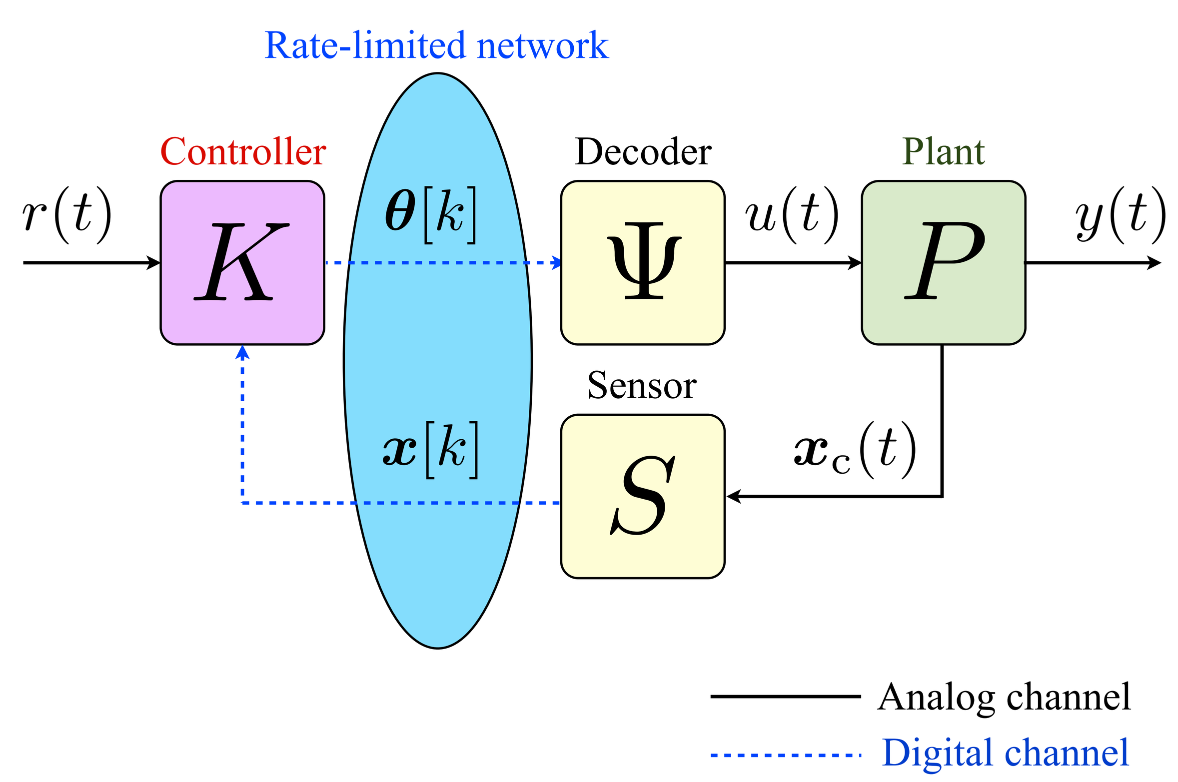
We investigate the use of *compressive sampling* for *networked feedback control systems*. The method proposed serves to compress the control vectors which are transmitted through rate-limited channels without much deterioration of control performance. The control vectors are obtained by an *ℓ^1-ℓ^2 optimization*, which can be solved very efficiently by FISTA (Fast Iterative Shrinkage-Thresholding Algorithm). Simulation results show that the proposed sparsity-promoting control scheme gives *a better control performance* than a conventional energy-limiting L^2-optimal control.

Sparsity in Control

- The plant P is located *away* from the controller K. Remote control robots, aircrafts, and vehicles are examples.

- We should use a *rate-limited network* (e.g., wireless communication network, or the Internet) for sending the control signal (vector $\boldsymbol{\theta}[k]$) to the plant P.
- Sparse vectors can be *encoded more effectively than dense vectors* obtained by usual energy-limiting optimization as in Linear Quadratic (LQ) Optimal Control. For example, we quantize the non-zero coefficients in $\boldsymbol{\theta}[k]$ and, in addition, send information about the coefficient locations.
- In many cases, sparse vectors have *small ℓ^1 norm*. This leads to *robustness* against uncertainty (or perturbation) in the plant model (c.f. small gain theorem).

Networked Control System



Control Problem

The control objectives:

1. Good tracking performance: $r(t) \approx y(t)$
2. Sparse control vector: $\|\boldsymbol{\theta}[k]\|_0 \ll N$

Notation: Let $T > 0$ be the sampling period of the control system. For a continuous-time signal v on $[0, \infty)$, we denote by v_k, $k = 0, 1, \ldots$, the restriction of v to $[kT, kT + T)$, that is,

$$v_k(t) := v(kT + t),\ t \in [0, T),\ k = 0, 1, \ldots$$

Assumption: Let M be a given positive integer and define the signal subspace

$$\begin{aligned} V_M &:= \operatorname{span}\{\psi_{-M}, \ldots, \psi_M\} \\ \psi_m(t) &:= \tfrac{1}{\sqrt{T}} \exp(\mathrm{j}\omega_m t),\ t \in [0, T), \\ \omega_m &:= 2\pi m/T. \end{aligned}$$

We assume that $r_k \in V_M$ and $u_k \in V_M$. That is, the reference signal r and the control signal u are *band-limited* up to the Nyquist frequency $\omega_M = 2\pi M/T$ on each time interval $[kT, kT + T)$.

Solution by Random Sampling and ℓ^1-ℓ^2 Optimization

Problem Formulation

Given reference signal $r_k \in V_M$, $k = 0, 1, \ldots$, find the control $u_k \in V_M$ (or the vector $\boldsymbol{\theta}[k]$) that minimizes

$$J := \int_0^T |y_k(t) - r_k(t)|^2\,\mathrm{d}t + \mu\|\boldsymbol{\theta}[k]\|_0 \qquad (*)$$

where $\boldsymbol{\theta}[k]$ is the Fourier coefficient vector:

$$u_k(t) = \sum_{m=-M}^{M} \theta_m[k]\psi_m(t)$$

Random Sampling

To obtain the vector $\boldsymbol{\theta}[k]$, one has to sample the continuous-time signal r_k. Since r_k is assumed to be band-limited up to $\omega_M = 2\pi M/T$ [rad/sec], one can sample r_k at a sampling frequency higher than $2\omega_M$, by Shannon's sampling theorem.

However, if M is very large, it may take very long time to compute the optimal vector. Therefore, we adopt *random sampling* as used in compressed sampling.

The cost function J in (*) is then reduced to

$$J = \|\Phi\boldsymbol{\theta}[k] - \boldsymbol{\alpha}[k]\|_2^2 + \mu\|\boldsymbol{\theta}[k]\|_0,$$

where $\Phi \in \mathbb{R}^{K\times N}$ $(K < N)$ and $\boldsymbol{\alpha}[k] \in \mathbb{R}^K$ are independent of $\boldsymbol{\theta}[k]$.

ℓ^1-ℓ^2 Optimization

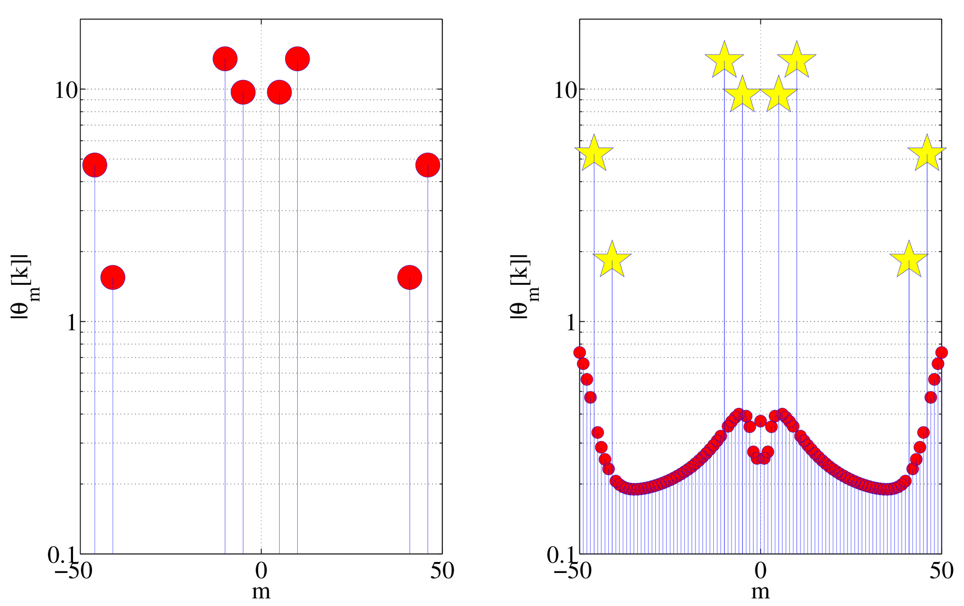
The optimization is executed in the feedback loop, and hence the optimal vector should be computed *as fast as possible*, since a delay leads to instability of the feedback system. Therefore, we relax the cost function as

$$J = \|\Phi\boldsymbol{\theta}[k] - \boldsymbol{\alpha}[k]\|_2^2 + \mu\|\boldsymbol{\theta}[k]\|_1,$$

and solve this ℓ^1-ℓ^2 optimization via *FISTA*.

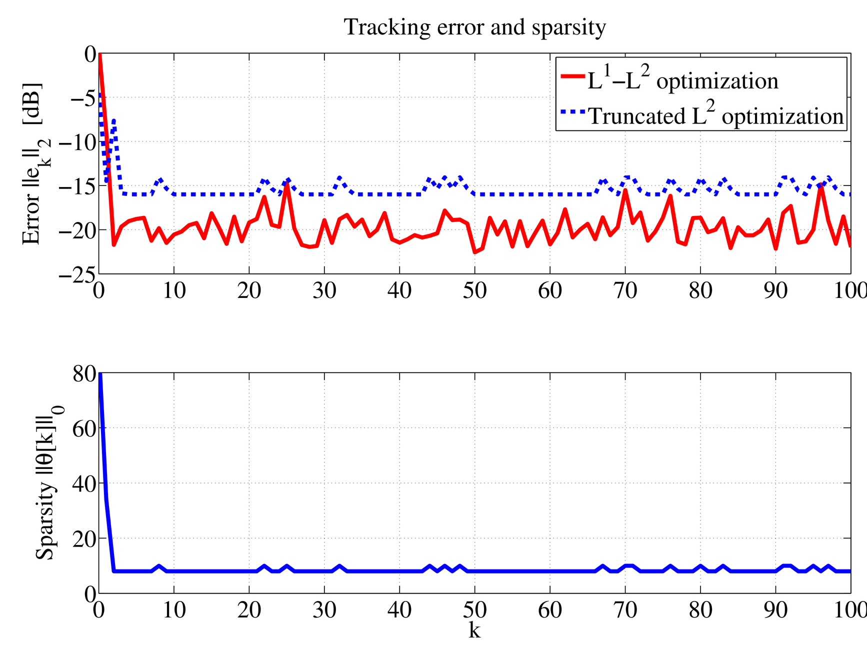
Simulation Results

Control vector $\boldsymbol{\theta}[k]$



Left: proposed, Right: conventional

Control performance $\|e_k\|_2$ and sparsity



The sparsity of the control vector obtained by the proposed method is about 8 out of 101 in the steady state.

Conclusion

We have studied the use of compressive sampling for feedback control systems with rate-limited communication channels. Simulation studies indicate that the method proposed can effectively compress the signals transmitted. Future work could include further investigation of bit-rate issues and the study of closed loop stability.

References

[A] M. Nagahara and D.E. Quevedo, *IFAC World Congress*, pp. 84–89, 2011.

[B] M. Nagahara, T. Matsuda, and K. Hayashi, *IEICE Trans. Fundamentals*, Vol. E95-A, No. 4, 2012.