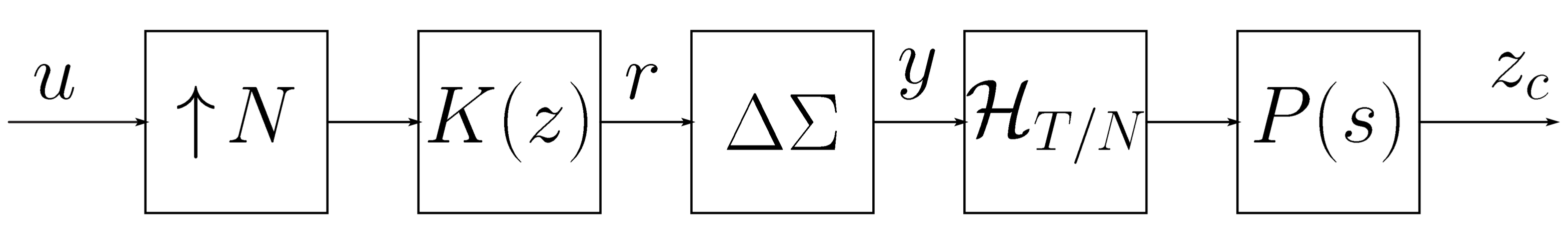
Oversampling $\Delta\Sigma$ DA Converters



u: digital signal ($1/T$ [Hz] and b [bits])

$\uparrow N$: upsampler

$K(z)$: interpolation filter to be designed

$\Delta\Sigma$: $\Delta\Sigma$ modulator

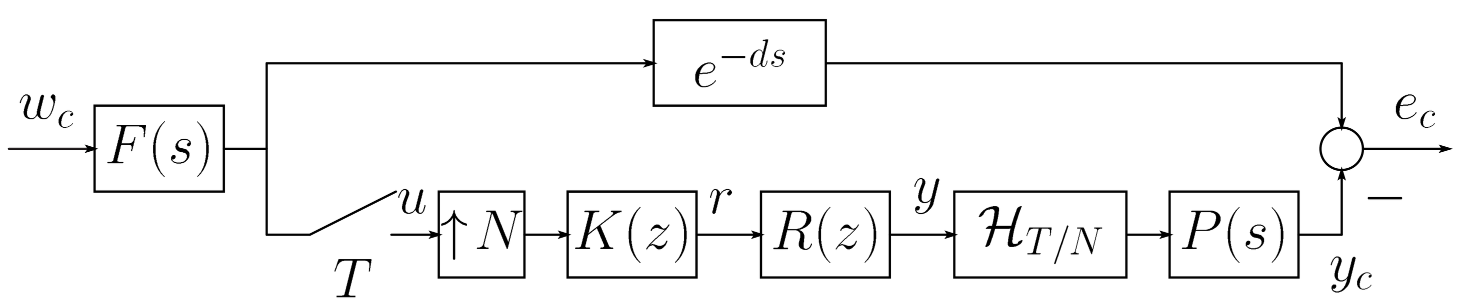
$\mathcal{H}_{T/N}$: hold device with sampling time T/N [sec]

$P(s)$: analog lowpass filter

z_c: reconstructed analog signal

Our objective here is to design the interpolation filter $K(z)$ to interpolate samples taking account of the analog performance.

Interpolator Design



w_c: analog signal in L^2

$F(s)$: frequency characteristic of analog input signal

Design Problem

Given a stable, strictly causal $F(s)$, stable, causal $P(s)$, stable, strictly causal $R(z)$, upsampling factor N, delay d, sampling time T, find $K(z)$ which minimizes

$$\|\mathcal{E}\|_\infty := \sup_{w_c \in L^2} \frac{\|e_c\|_{L^2}}{\|w_c\|_{L^2}}.$$

The optimal filter $K(z)$ can be obtained via sampled-data H^∞ optimization.

Design Example

Design Parameters

sampling time: $T = 1$

upsampling ratio: $N = 8$

reconstruction delay: $d = 8$

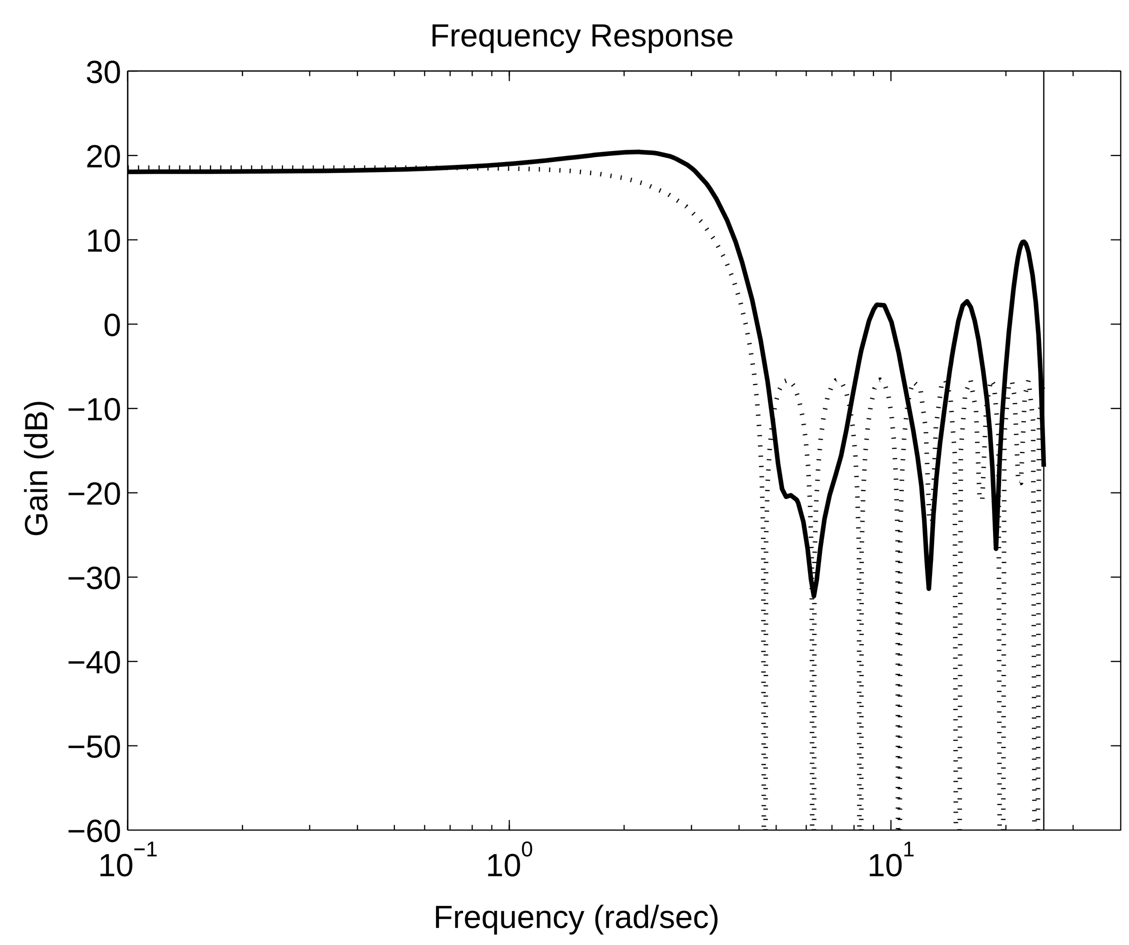
analog filters:

$$P(s) = \frac{1}{\{(T/\pi)s + 1\}^2}$$

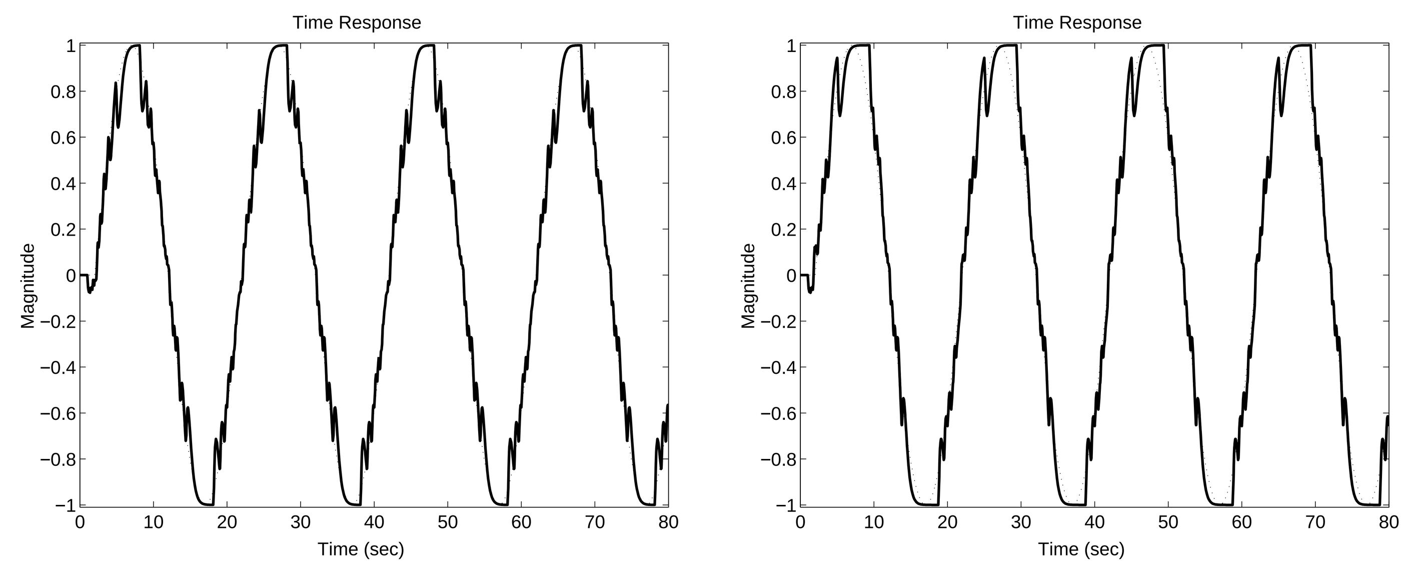
$$F(s) = \frac{1}{\{(T_F/\pi)s + 1\}\{(0.1T_F/\pi)s + 1\}}, \quad T_F = 22.05$$

quantizer:

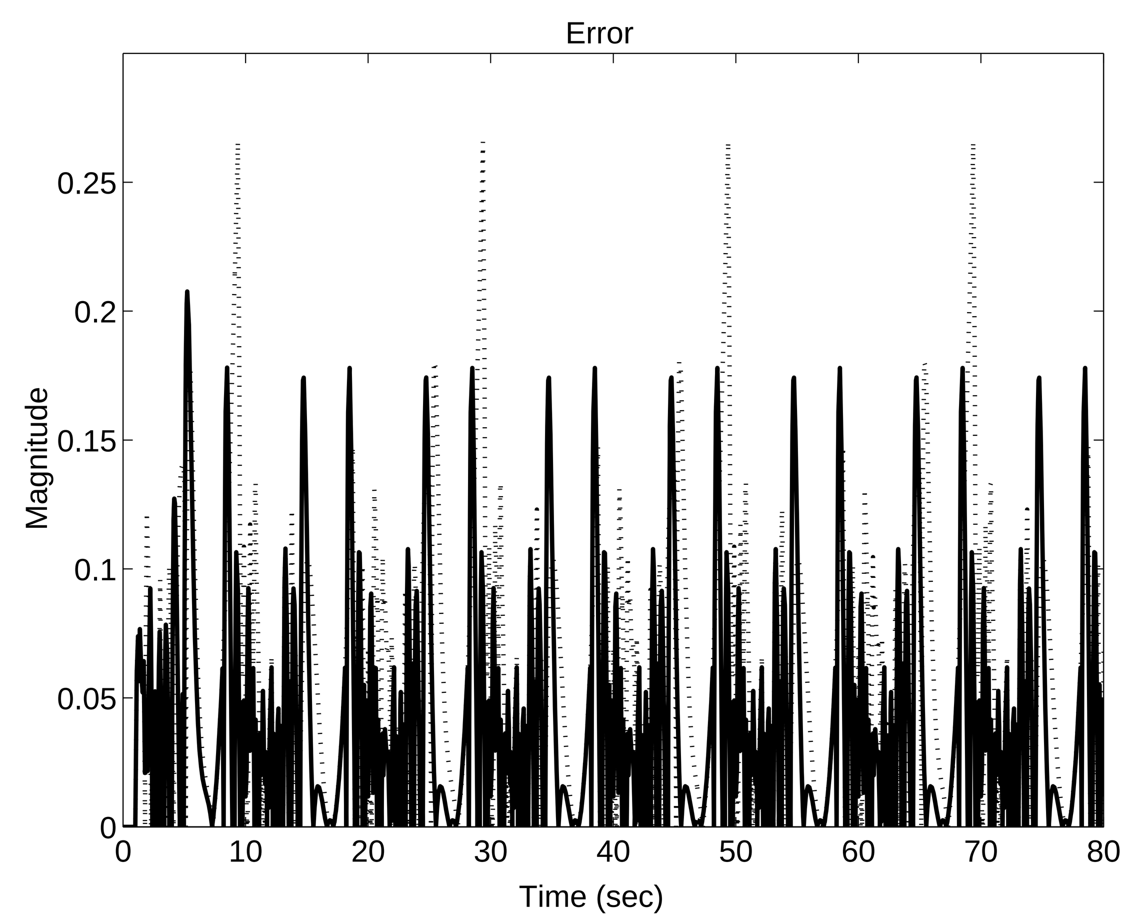
$$Q(\psi) = \mathrm{sgn}(\psi) = \begin{cases} 1, & \psi \geq 0, \\ -1 & \psi < 0. \end{cases}$$



Interpolation filter $K(z)$ (solid: sampled-data H^∞ optimal, dotted: equiripple design)



Time response against a sinusoidal wave $u[k] = \sin(0.1\pi k)$ (left: proposed, right: conventional)



Absolute error (solid: proposed, dotted: conventional)

Conparison of error

	Proposed design	Conventional design
$\|e_c\|_\infty$	2.08×10^{-1}	2.67×10^{-1}
$\|e_c\|_2$	5.68×10^{-1}	7.21×10^{-1}
$\mathrm{RMS}(e_c)$	6.34×10^{-2}	8.06×10^{-2}

In the table, RMS is the root-mean-square values defined as follows: For fixed $T_f > 0$, $\mathrm{RMS}(e_c) := \left\{\frac{1}{T_f}\int_0^{T_f} |e_c(t)|^2 dt\right\}^{\frac{1}{2}}$.

Conclusion

We have proposed a new design method for $\Delta\Sigma$ modulators and oversampling $\Delta\Sigma$ DA converters via H^∞ optimization. We have presented design examples and shown the advantages of the present method.