Design of Oversampling $\Delta\Sigma$ DA Converters via H^∞ Optimization

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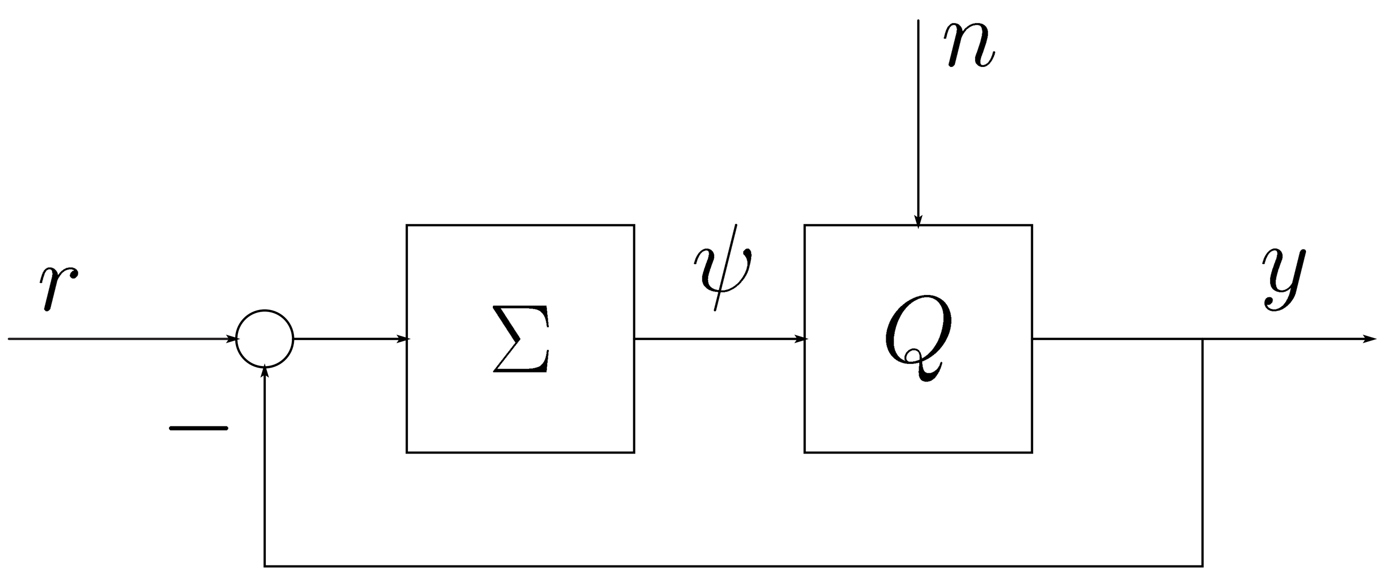
Abstract

In this paper, we propose a new method for designing oversampling $\Delta\Sigma$ DA converters via H^∞ optimization. The design consists of two steps. One is that for $\Delta\Sigma$ modulators. In $\Delta\Sigma$ modulators, the accumulator $1/(z-1)$ is conventionally used in a feedback loop to shape quantization noise. In contrast, we give all stabilizing loop filters for the modulator, and propose an H^∞ design to shape the frequency response of the noise transfer function (NTF). The other is a design for interpolation filters in oversampling DA converters. While conventional designs are executed in the discrete-time domain, we take account of the characteristic of the original analog signal by using sampled-data H^∞ optimization. A design example is presented to show that our design is superior to conventional ones.

$\Delta\Sigma$ Modulator



r: Input signal to be modulated

y: Modulated signal

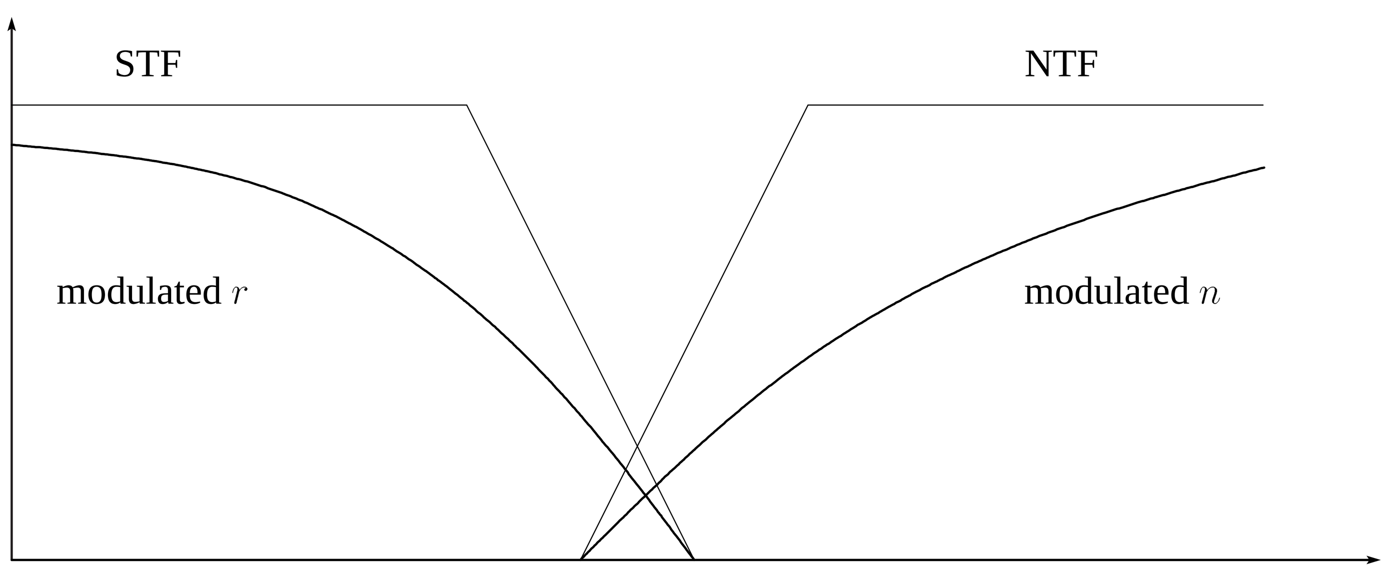
n: Quantization noise

$R(z): r \to y$: Signal Transfer Function (STF)

$H(z): n \to y$: Noise Transfer Function (NTF)

Quantization Noise Shaping

If r contains few high frequency components, we can separate the noise n from the output signal y by a allpass or lowpass STF and a highpass NTF.



Example

Conventional first order $\Delta\Sigma$ Modulator.

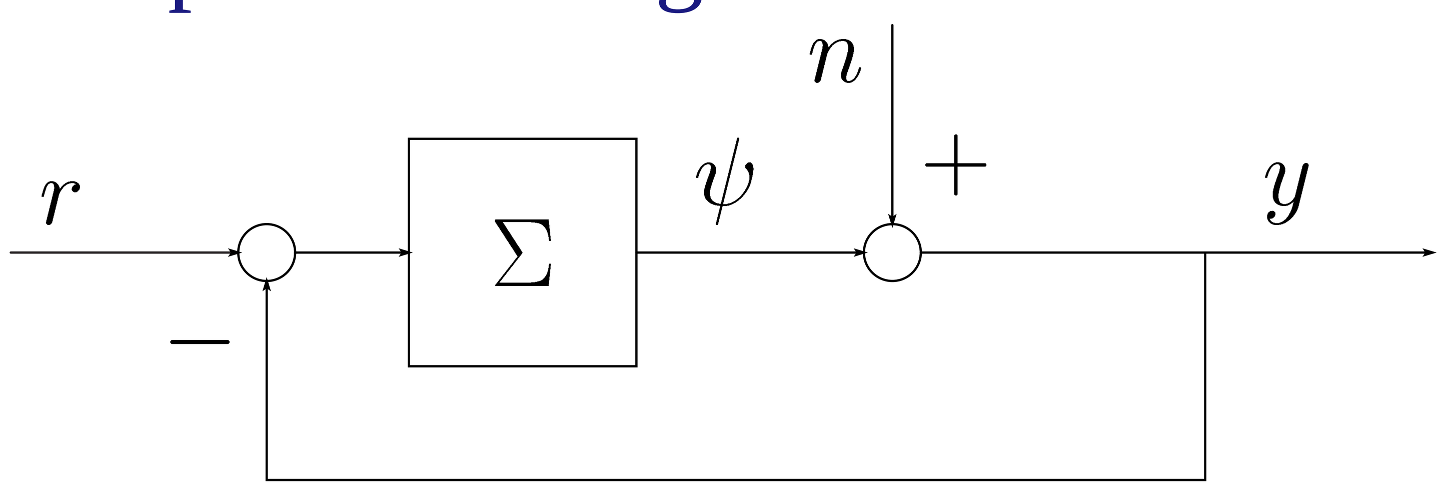
$\Sigma(z) = \frac{1}{z-1}$

$y = \frac{\Sigma(z)}{1+\Sigma(z)} r + \frac{1}{1+\Sigma(z)} n = z^{-1} r + (1 - z^{-1}) n$

STF: $R(z) = z^{-1}$: Allpass

NTF: $H(z) = 1 - z^{-1}$: Highpass

Loop Filter Design



Lemma 1

The above system is well-posed and internally stable if and only if

$$\Sigma(z) \in \left\{ \frac{R(z)}{1 - R(z)} : R(z) \text{ is stable and strictly causal} \right\}$$

Design Problem

Given a stable transfer function $H_{des}(z)$ (desired NTF) and a stable weighting function $W(z)$, find $H(z)$ with $H(\infty) = 1$ (required for well-posedness) which minimizes

$$J(H) = \|(H - H_{\rm des})W\|_\infty.$$

Design by LMI

Assume $H(z)$ is FIR, that is,

$$H(z) = \sum_{k=1}^{N} a_k z^{-k}.$$

Then, by using the *bounded real lemma*, the optimization is reducible to a linear matrix inequality (LMI) with respect to a matrix variable and the coefficients $a_1, \ldots, a_N$.

Zeros of NTF

Define $n_H(z) := z^N - \Sigma_{k=1}^N a_k z^{N-k}$ (the numerator of $H(z)$). Then, $H(z)$ has at least M zeros at $z = z_0$ if and only if

$$\left. \frac{d^k n_H(z)}{dz^k} \right|_{z=z_0} = 0, k = 0, 1, \ldots, M-1 \text{ (Linear constraints).}$$

Stability Constraints

We can add a stability condition considering nonlinear behaviors of $\Delta\Sigma$ modulators by

$$\|H\|_\infty < C$$

where $C = 1.5$ for binary quantizer (*Lee Criterion*), or

$$C = \frac{1}{(2\nu + 1)} (M + 2 - \|r\|_\infty)$$

for M-step quantizer with ν-th order $H(z)$. This is also reducible to an LMI.