Robust Repetitive Control by Sampled-data H^∞ Filters

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***Abstract*— In this article, we study digital repetitive control to track continuous-time periodic references. Sampled-data control gives a very natural platform for studying digital repetitive control, and in particular, sampled-data H^∞ filtering is particularly suitable for this. The link is provided by introducing internal model control. This leads to a close connection between robust repetitive control and sampled-data H^∞ noise reduction, or regularization. With this connection, it becomes possible to employ a variety of filtering techniques, such as multirate filtering, for robust digital repetitive control. Design examples show the effectiveness of our method in robustness in digital repetitive control.**

I. INTRODUCTION

Repetitive control is a control scheme which is designed for tracking periodic reference signals or for rejecting periodic disturbances [1]. This method was first introduced in the control of magnetic power supply for proton synchrotron [2]. Since then, a number of theoretical and industrial studies have been made on repetitive control, see surveys [3], [4] and references therein.

In view of the internal model principle [5], [6], repetitive control system must include the periodic signal generator

$$Q_{\mathrm{c}}(s) = \frac{\mathrm{e}^{-Ls}}{1-\mathrm{e}^{-Ls}}$$

in the feedback loop. This enables us to track *arbitrary* periodic references of period L with zero steady-state error. There however arise two problems; stability and implementability.

The stability problem is due to the fact that the repetitive control system is a neutral delay-differential system. By this nature, the control system cannot be exponentially stable if the transfer function of the plant is strictly proper [1], [6]. To remedy this, modified repetitive control [7], [1] was proposed, in which the repetitive controller $Q_{\mathrm{c}}(s)$ is replaced by

$$Q_{\mathrm{c}}^{\mathrm{mod}}(s) = \frac{F_{\mathrm{c}}(s)\mathrm{e}^{-Ls}}{1-F_{\mathrm{c}}(s)\mathrm{e}^{-Ls}} \tag{1}$$

where $F_{\mathrm{c}}(s)$ is a lowpass filter with a cutoff frequency ω_{c}. This repetitive control enables us to exponentially stabilize strictly proper plants, and to well track reference signals up to the cutoff frequency ω_{c}.

The modified repetitive control provides a reasonable scheme for general plants; however, there remains the second problem: implementability. This is due to the infinite dimensionality of the delay e^{-Ls}, which is difficult to implement with an analog device. For this problem, controller discretization is an effective solution. In this approach, the delay e^{-Ls} is conventionally replaced by a sampled-data system $\mathbf{H}_h z^{-m}\mathbf{S}_h$, where $\mathbf{H}_h$ and $\mathbf{S}_h$ are, respectively, the zero-order hold and the ideal sampler with sampling period h, and m is a positive integer satisfying $L = mh$. This leads to digital repetitive control [8] with repetitive controller

$$Q_{\mathrm{d}}(z) = \frac{z^{-m}}{1-z^{-m}}.$$

This is the signal generator for arbitrary *discrete-time* periodic signals of an integer period m, by which the control system can achieve perfect tracking[1] for the periodic signals on the sampling instants.

While this appears to be a reasonable compromise, it inevitably induces intersample ripples, which clearly limits the tracking capability. Although a partial solution may be obtained by introducing a generalized hold [9] or a multirate control [10], one wants to have an overall performance estimate and control of such intersample ripples. This can be effectively handled by modern *sampled-data control theory* [11], [12], and there are indeed several works on sampled-data repetitive control: [13], [14], [15], [16], [17].

While motivated by these works, we here propose another novel repetitive control scheme viewed from the viewpoint of digital filter design. This approach makes use of the internal model control (IMC) [18], instead of the modified repetitive control, and gives an effective solution to the following problems discussed above:

- stability,
- implementability,
- and intersample ripples.

The principle of IMC is in incorporating the model in the controller to handle the intrinsic difficulty in the plant. In our case, the plant may be strictly proper and non-minimum phase (i.e., difficult to control), and hence we place the model into our repetitive controller.

The objective of the present article is to show that by using the IMC structure, sampled-data H^∞ filters can effectively solve the three problems simultaneously listed above. This filter was first proposed in [19], and since then a number

[1] Perfect tracking means that the steady-state tracking error is zero.

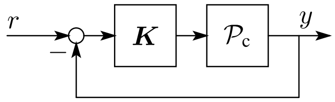

Fig. 1. Digital repetitive control system: $\boldsymbol{K}$ is a sampled-data repetitive controller, $\mathcal{P}_{\mathrm{c}}$ is a stable plant which can have unmodeled dynamics.

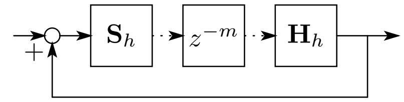

Fig. 2. Conventional digital repetitive controller.

of works have appeared on various digital signal processing problems, such as sampling rate converters [20], [21], optimal wavelet expansion [22], fractional delay filters [23], JPEG noise reduction [24] and $\Delta\Sigma$ converters [25]. See also a survey paper [26].

The sampled-data H^∞ filter is a digital one, which is designed to approximate the time delay e^{-Ls} with respect to the H^∞ norm of the sampled-data error system. It is noted [26] that the conventional Shannon reconstruction causes high-frequency errors due to the Gibbs phenomenon. This is because the conventional filter is designed to *perfectly* reconstruct the original signal within the Nyquist frequency. Without confining ourselves to this idea of perfect reconstruction (however below the Nyquist frequency), it is possible to obtain a better overall performance by using sampled-data H^∞ filters.

We have proposed this approach in [27], but the robustness was not discussed there. In this article, we also show that robust repetitive control can be described by sampled-data H^∞ noise reduction or *regularization*. Regularization is used in machine learning [28] or inverse problems [29], [30]. Using this technique, we can well control a trade-off between fitting of the given input data and robustness against model uncertainties. In regularization, robustness can be achieved by reducing a norm of the solution. In the same vein, we apply regularization to achieve robustness in repetitive control.

The organization of this article is as follows. In Section II, we formulate the problem of repetitive controller design. We introduce IMC-based digital repetitive control in Section III, where we discuss the relation between repetitive control and sampled-data H^∞ filtering. We also consider robust stability there. Section IV illustrates design examples to show effectiveness of our method. Section V concludes this article.

A. Notation

The subscripts 'c' and 'd' indicate, respectively, "continuous-time" and "discrete-time." We use boldface letters (e.g., $\boldsymbol{K}$) for sampled-data systems (i.e., systems including both continuous- and discrete-time signals); in particular, $\mathbf{S}_h$ and $\mathbf{H}_h$ are the ideal sampler with period h and the zero-order hold with period h, respectively. L^2 denotes the square integrable functions on $\mathbb{R}_+$ (the strictly positive real numbers), and $\mathcal{B}(L^2)$ the linear bounded operators in L^2.

II. Digital Repetitive Control Problem

Here we formulate a repetitive control problem. Consider the control system shown in Fig. 1. In this figure, $\mathcal{P}_{\mathrm{c}}$ is a plant to be controlled. Throughout this article, we assume that the plant $\mathcal{P}_{\mathrm{c}}$ is stable, or already stabilized by another controller. The plant can be perturbed due to uncertainty in modeling. The controller $\boldsymbol{K}$ we design is a sampled-data system. In this setting, the control system should achieve

- (robust) stability,
- and tracking reference r of period L.

As mentioned above, if the relative degree of $\mathcal{P}_{\mathrm{c}}$ is nonzero, perfect tracking and stability are incompatible. Thus, we must relax the strict requirement of perfect tracking. Our problem is formulated as follows.

Problem 1: Find a (robustly) stabilizing controller $\boldsymbol{K}$ such that the sensitivity function $\boldsymbol{S} = (I + \mathcal{P}_{\mathrm{c}}\boldsymbol{K})^{-1}$ is sufficiently small at frequencies

$$\omega_n := \frac{2\pi n}{L}, \quad n = 0, 1, 2, \ldots, N$$

where N is a given positive integer.

This is the main idea of modified repetitive control. We however do not adopt the structure of (1), but introduce an internal model control structure.

III. Repetitive Control with IMC Structure by Sampled-data H^∞ Filters

A. Sensitivity Optimization

The conventional digital repetitive controller is shown in Fig. 2. This is an approximation of the continuous-time repetitive controller $Q_{\mathrm{c}}(s) = \mathrm{e}^{-mhs}/(1 - \mathrm{e}^{-mhs})$ assuming that m is a positive integer satisfying $L = mh$. By the equation $\mathbf{S}_h\mathbf{H}_h = I$ (identity) [11], the digital repetitive controller is given by $\mathbf{H}_h Q_{\mathrm{d}} \mathbf{S}_h$ where $Q_{\mathrm{d}}(z) = z^{-m}/(1 - z^{-m})$. This Q_{d} is the signal generator for arbitrary discrete-time periodic signals of period m. By the discrete-time version of the internal model principle, the repetitive control system with Q_{d} can perfectly track arbitrary m-periodic references. Note however that this can lead to intersample ripples, since this accomplishes tracking only at sampled points. Here we propose an alternative to Q_{d} via the internal model control (IMC) scheme and sampled-data H^∞ filters. The proposed repetitive control system is shown in Fig. 3. In this figure, $\mathcal{P}_{\mathrm{c}}$ is a stable continuous-time plant and P_{c} denotes a model of $\mathcal{P}_{\mathrm{c}}$. The system F_{c} is assumed to be stable and strictly proper. This F_{c} is usually called an anti-aliasing filter. This however is not necessarily required to have a cutoff frequency lower than the Nyquist frequency π/h. This is introduced to make the feedback system to be L^2-bounded (see [11]).

The sampled-data H^∞ optimal filter is an approximation of the continuous-time delay e^{-Ls} taking account of intersample errors by using sampled-data H^∞ control theory. This fact can be used in our IMC repetitive control. Assume for the moment that $\mathcal{P}_{\mathrm{c}} = P_{\mathrm{c}}$, and the sampling frequency

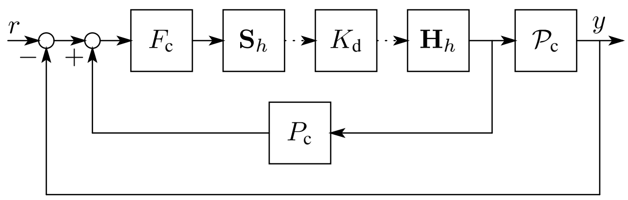

Fig. 3. Sampled-data repetitive control system with IMC structure.

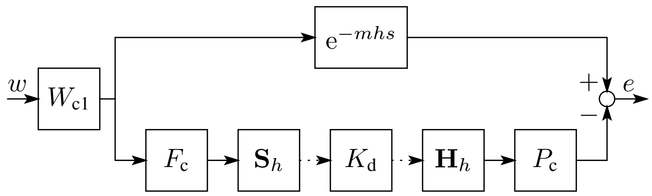

Fig. 4. Error system for approximation of e^{-mhs} where m is a positive integer.

$2\pi/h$ is non-pathological [11]. Then, by using the Youla parameterization, the feedback system is stable if and only if the digital controller K_d is stable. If K_d is stable, the sensitivity function is given by

$$\boldsymbol{S} = 1 - P_c \mathbf{H}_h K_d \mathbf{S}_h F_c.$$

To solve our problem, we use a weighting function W_{c1} which is stable and lowpass. By using this weighting function, we optimize the following objective function

$$J_1(K_d) = \|(e^{-mhs} - P_c \mathbf{H}_h K_d \mathbf{S}_h F_c) W_{c1}\|_\infty \tag{2}$$

where the norm is the L^2-induced norm of the sampled-data error system which is equivalent to the H^∞ norm of the lifted system [11]. Roughly speaking, if this is sufficiently small, then we have $P_c \mathbf{H}_h K_d \mathbf{S}_h F_c \approx e^{-mhs}$ in low frequencies determined by the cut-off frequency of W_{c1}. Then the sensitivity function $\boldsymbol{S}$ can be small at $\{j\omega_n\}_{n=1,2,\ldots,N}$. The corresponding error system is shown in Fig. 4. The optimal filter, called sampled-data H^∞ filter, can be obtained numerically by the fast-sampling method [19], [31] or H^∞ discretization [32].

B. Robust Stability Conditions

We consider uncertainty in the plant $\mathcal{P}_c$. Let $\mathcal{D} := \{\Delta \in \mathcal{B}(L^2) : \|\Delta\|_\infty \le 1\}$, where $\mathcal{B}(L^2)$ is the set of the linear bounded operators in L^2. We have the following lemma [11].

Lemma 1: Assume K_d is stable.

1) The control system shown in Fig. 3 for any $\mathcal{P}_c$ of the form (multiplicative perturbations) $\mathcal{P}_c = P_c(1+\Delta W_{c2})$ where $\Delta \in \mathcal{D}$ and W_{c2} is a stable weighting function, is internally stable if
$$\|P_c \mathbf{H}_h K_d \mathbf{S}_h F_c W_{c2}\|_\infty < 1. \tag{3}$$
2) The control system shown in Fig. 3 for any $\mathcal{P}_c$ of the form (additive perturbations) $\mathcal{P}_c = P_c + \Delta W_{c2}$ where $\Delta \in \mathcal{D}$, and W_{c2} is a stable weighting function, is internally stable if $\|\mathbf{H}_h K_d \mathbf{S}_h F_c W_{c2}\|_\infty < 1$.

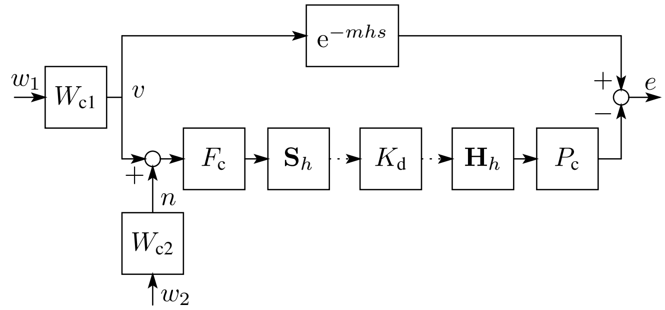

Fig. 5. Error system for robust repetitive controller.

C. Robust Control

Here we formulate a robust controller design problem in our repetitive control framework. For simplicity, we consider only the case of multiplicative perturbations, although an almost parallel discussion applies to the case of additive perturbations.

To relate the robust stability condition (3) with our objective function J_1 in (2), we define the following two-block objective function

$$J_2(K_d) = \left\| \begin{bmatrix} (e^{-mhs} - P_c \mathbf{H}_h K_d \mathbf{S}_h F_c) W_{c1} \\ P_c \mathbf{H}_h K_d \mathbf{S}_h F_c W_{c2} \end{bmatrix} \right\|_\infty \tag{4}$$

where W_{c1} and W_{c2} are stable weighting functions. It should be noted that the weighting functions W_{c1} and W_{c2} must be chosen considering the trade-off between performance and robustness. As mentioned above, our goal is to track periodic signals in low frequencies, and hence the weighting function W_{c1} is chosen to be a lowpass filter. On the other hand, if W_{c2} is also a lowpass filter, this will fundamentally limit the tracking performance since the filter cannot have enough gain in low frequencies. Usually the noise n have more energy in high frequencies, so we choose a highpass W_{c2}, in particular, to have the property $W_{c2}(0) = 0$. We also assume that the norm $\|W_{c2}\|_\infty$ be much smaller than $\|W_{c1}\|_\infty$ since a large W_{c2} can seriously degrade the tracking performance. See the example in IV-B below.

The corresponding block diagram for designing a robust controller is shown in Fig. 5. This diagram provides an interpretation of the design as noise reduction. Continuous-time noise n is added before A/D (Analog-to-Digital) conversion (i.e., $\mathbf{S}_h F_c$), and the filter K_d attempts to reduce the noise n and reconstruct the input signal v. This interpretation gives a recipe for designing the analog device F_c (anti-aliasing filter). That is, if $W_{c2}(s)$ is a highpass filter with cutoff frequency ω_{c2}, then $F_c(s)$ can be chosen as a lowpass filter with the same (or a lower) cutoff frequency. This however can decrease the tracking performance in high frequencies.

Another interpretation of the robust controller design is *regularization* [28], [29], [30]. The design in the previous subsection to minimize (2) is a kind of inverse problem; we seek a sampled-data (delayed) inverse system $\mathbf{H}_h K_d \mathbf{S}_h$ of the uncertain continuous-time system "$F_c P_c$" whose inverse cannot be stable and causal. This type of inverse problem

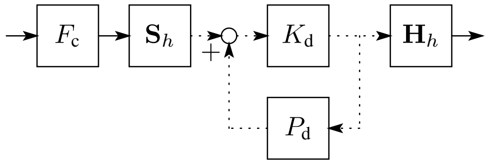

Fig. 6. Digital implementation of sampled-data repetitive controller: P_{d} is the step-invariant discretization of P_{c}.

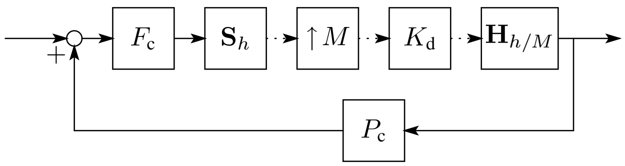

Fig. 7. Multirate repetitive controller with upsampler $\uparrow M$.

is called *ill-posed*, for which regularization is a powerful tool. Regularization limits the norm of an inverse system to obtain a robust inverse against uncertainty of a target system or data. We can see that the objective function (4) limits the H^∞ norm of $P_{\mathrm{c}}\mathbf{H}_h K_{\mathrm{d}}\mathbf{S}_h F_{\mathrm{c}}$ with weighting function (regularization parameter) $W_{\mathrm{c}2}$. Then the gain of the resulting filter K_{d} is reduced in high frequencies determined by $W_{\mathrm{c}2}$. See the design example in IV-B.

The optimal controller which minimizes (4) can be computed numerically by fast-sampling method [31] or H^∞ discretization [32].

D. Implementation

Our sampled-data controller in Fig. 3 includes both a discrete-time controller K_{d} and a continuous-time plant model P_{c}. To implement this with a digital device, we equivalently convert the sampled-data system into a discrete-time system. To this end, we introduce the step-invariant transformation [11], $P_{\mathrm{d}} = \mathbf{S}_h F_{\mathrm{c}} P_{\mathrm{c}} \mathbf{H}_h$. This system is a linear time-invariant discrete-time system, by which our controller is transformed into

$$\boldsymbol{K} = \mathbf{H}_h \left(\frac{K_{\mathrm{d}}}{1 - P_{\mathrm{d}} K_{\mathrm{d}}} \right) \mathbf{S}_h F_{\mathrm{c}}.$$

This controller is illustrated in Fig. 6.

E. Multirate Filtering and Control

It is pointed out in [10] that in digital repetitive control systems, the period of the hold device can be shorter than that of the sampler. Considering this advantage, we can adopt a multirate controller shown in Fig. 7 instead of the controller in Fig. 3 In this system, $\uparrow M$ is the upsampler [33] defined by

$$\uparrow M : \{x[k]\}_{k=0}^{\infty} \mapsto \{x[0], \underbrace{0, \ldots, 0}_{M-1}, x[1], 0, \ldots\}.$$

Assuming that $\mathcal{P}_{\mathrm{c}} = P_{\mathrm{c}}$, the controller K_{d} is designed to optimize the following objective function

$$J_3(K_{\mathrm{d}}) = \|\{\mathrm{e}^{-mhs} - P_{\mathrm{c}}\mathbf{H}_{h/M} K_{\mathrm{d}}(\uparrow M)\mathbf{S}_h F_{\mathrm{c}}\} W_{\mathrm{c}1}\|_\infty. \quad (5)$$

This is a multirate filtering problem addressed in [21]. The optimal filter can be numerically obtained by using the discrete-time lifting [11], see [21]. Digital implementation as shown in Fig. 6 is also obtained by discrete-time lifting.

Robust controller as proposed in III-C can also be designed by the following objective function

$$J_4(K_{\mathrm{d}}) = \left\| \begin{bmatrix} (\mathrm{e}^{-mhs} - P_{\mathrm{c}}\mathbf{H}_{h/M} K_{\mathrm{d}}(\uparrow M)\mathbf{S}_h F_{\mathrm{c}}) W_{\mathrm{c}1} \\ P_{\mathrm{c}}\mathbf{H}_{h/M} K_{\mathrm{d}}(\uparrow M)\mathbf{S}_h F_{\mathrm{c}} W_{\mathrm{c}2} \end{bmatrix} \right\|_\infty . \quad (6)$$

The optimal K_{d} can be numerically obtained by fast-sampling method [31] or H^∞ discretization [32], with discrete-time lifting.

IV. Design Examples

In this section, we show design examples. The nominal plant P_{c} here is

$$P_{\mathrm{c}}(s) = \frac{s-1}{2s^3 + 3s^2 + 4s + 5}.$$

The period L of reference signals is 20. The sampling period h is 1, and the delay steps in designing the H^∞ filter is $m = L/h = 20$. The first example shows efficiency of multirate structure of digital repetitive control. The second example discusses robustness of repetitive controllers.

A. Multirate Control

Here we show the effectiveness of multirate filters discussed in III-E. We assume that there is no perturbation in the plant, that is, $\mathcal{P}_{\mathrm{c}} = P_{\mathrm{c}}$.

The weighting function is a lowpass filter with cutoff frequency $\omega_{\mathrm{c}} = \pi/20$, that is,

$$W_{\mathrm{c}1}(s) = \frac{1}{T_{\mathrm{c}}s + 1}, \quad T_{\mathrm{c}} = \frac{20}{\pi}.$$

The lowpass filter F_{c} is chosen as

$$F_{\mathrm{c}}(s) = \frac{1}{Ts + 1}, \quad T = \frac{1}{100}.$$

With these parameters, we design two controllers; the sampled-data H^∞ optimal $K_{\mathrm{d1,opt}}$ minimizing J_1 in (2) and the multirate controller $K_{\mathrm{d3,opt}}$ minimizing J_3 in (5). The Bode gain plots of these controllers are shown in Fig. 8. In this figure, the inverse of P_{c} is also plotted. The controllers well simulate the shape of $|P_{\mathrm{c}}(\mathrm{j}\omega)^{-1}|$ in the low frequencies, while they are attenuated near the Nyquist frequencies (π and 4π). The shapes of the filters in the high frequencies result from the mini-max optimization with the trade-off between mimicking P_{c}^{-1} and reducing imaging [33] due to the zero-order hold $\mathbf{H}_h$ or $\mathbf{H}_{h/M}$ (and the upsampler $\uparrow M$ in the case of $M = 4$).

Then, by using these filters, we simulate the repetitive control for a rectangular reference of period $L = 20$. Fig. 9 shows the time response of the single-rate repetitive control system. This figure shows that there remain ripples around the edges of rectangle. The time response of the multirate repetitive control system is shown in Fig. 10. We can clearly see that the multirate structure substantially reduces the ripples in Fig. 9, and shows the effectiveness of the multirate

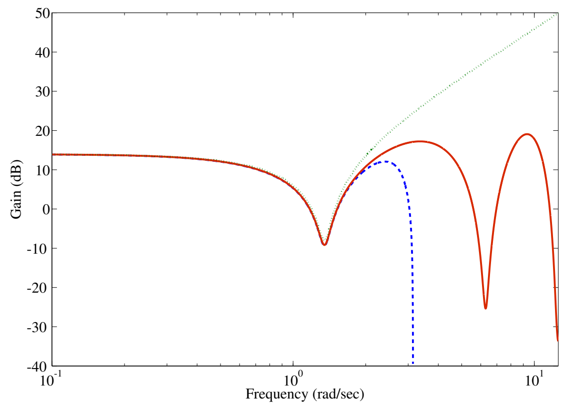

Fig. 8. Controllers; single-rate sampled-data H^∞ filter $K_{\mathrm{d1,opt}}$ (dash), multirate sampled-data H^∞ filter $K_{\mathrm{d3,opt}}$ (solid), and the inverse of P_{c} (dots).

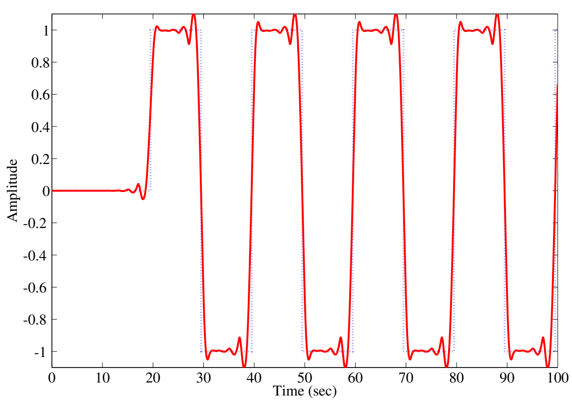

Fig. 9. Time response of single-rate sampled-data H^∞ optimal repetitive control system; delayed reference (dots) and output y (solid).

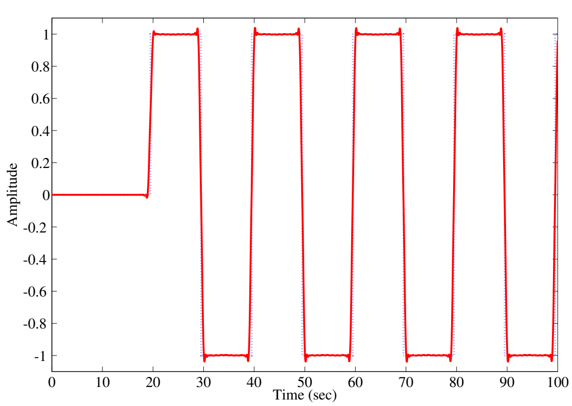

Fig. 10. Output of multirate ($M = 4$) sampled-data H^∞ optimal repetitive control system; delayed reference (dots) and output y (solid).

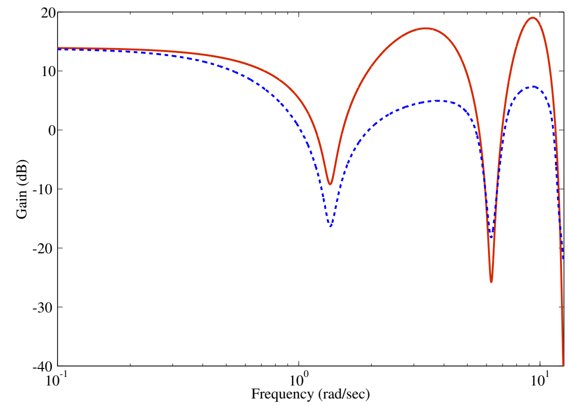

Fig. 11. Controllers; nominal controller $K_{\mathrm{d3,opt}}$ (solid) and robust controller $K_{\mathrm{d4,opt}}$ (dash).

controller when a faster hold device is available in the repetitive control system, although the sampler acts slowly. That is, multirate filtering is efficient for digital repetitive control.

B. Robust Control

Here we show an example of robust controller design discussed in III-C and III-E.

We design two controllers; multirate nominal controller $K_{\mathrm{d3,opt}}$ minimizing (5) with $M = 4$, and multirate robust controller $K_{\mathrm{d4,opt}}$ minimizing (6) with the weighting function

$$W_{\mathrm{c2}}(s) = 0.05 \cdot \frac{T_{\mathrm{c}} s}{T_{\mathrm{c}} s + 1}, \quad T_{\mathrm{c}} = \frac{20}{\pi}. \tag{7}$$

The number 0.05 is a trade-off parameter between performance and robustness, see the discussion in III-C. The other parameters are the same as ones in the previous subsection.

Fig. 11 shows the Bode gain plots of the designed filters. While the robust controller $K_{\mathrm{d4,opt}}$ is quite close to the nominal controller $K_{\mathrm{d3,opt}}$ in low frequencies, it has much lower gain in high frequencies. This attenuation is due to the effect of taking account of high-frequency uncertainty (7) in our robust design. This can also be interpreted as a result of regularization discussed in III-C. The difference between the peaks of two gains in high frequencies is about 10 dB.

Using these filters, we give simulation results of the repetitive control for a triangular reference signal of period $L = 20$. The plant is assumed to have a multiplicative perturbation defined by

$$\mathcal{P}_{\mathrm{c}}(s) = P_{\mathrm{c}}(s)\left(1 + \Delta(s)\right), \quad \Delta(s) = \frac{s-1}{s+1}.$$

Fig. 12 shows the tracking error of the repetitive control system with the nominal controller $K_{\mathrm{d2,opt}}$. Due to the effect of the plant perturbation, the error shows persistent response and decreases very slowly. On the other hand, by using the robust controller $K_{\mathrm{d4,opt}}$, the error decays much faster and exhibits a dramatic improvement in performance. These results show the present robust controller design is effective against uncertainty in the plant.

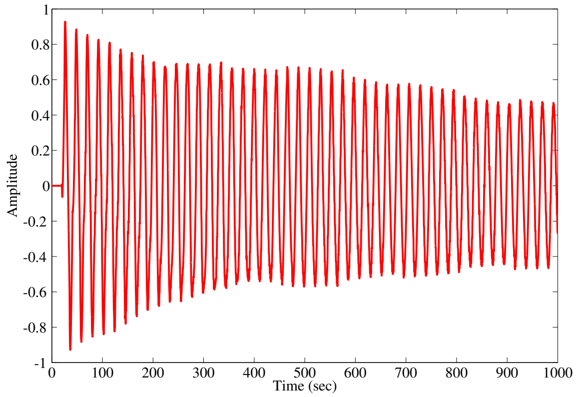

Fig. 12. Tracking error by nominal controller.

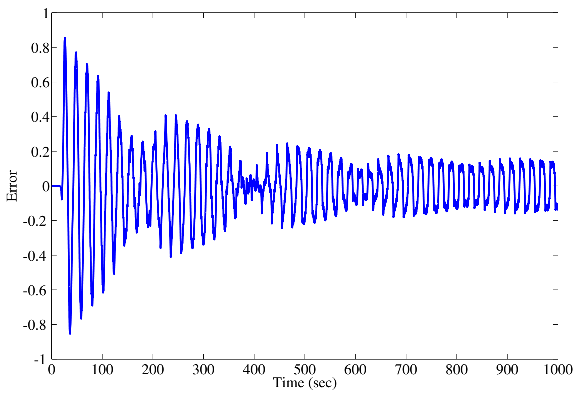

Fig. 13. Tracking error by robust controller.

V. Conclusion

In this article, we have proposed a novel repetitive control by sampled-data H^∞ filters. The controller has an IMC structure and is designed by solving a one-block sampled-data H^∞ control problem, which is equivalent to a sampled-data H^∞ filtering problem. We have also proposed a robust controller design, which is described by a two-block sampled-data H^∞ control problem. We have discussed a relation between the robust controller design and noise reduction or regularization. Design examples show the effectiveness of our method.

References

[1] S. Hara, Y. Yamamoto, T. Omata, and M. Nakano, "Repetitive control system: a new type servo system for periodic exogenous signals," *IEEE Trans. Automat. Contr.*, vol. 33, pp. 659–668, 1988.

[2] T. Inoue, M. Nakano, T. Kubo, S. Matsumoto, and H. Baba, "High accuracy control of a proton synchrotoron magnet power supply," in *Proc. of IFAC 8th World Congress*, 1981, pp. 216–221.

[3] Y. Yamamoto, "Learning control and related problems in infinite-dimensional systems," in *Essays on Control: Perspectives in the Theory and its Applications*, H. L. Trentelman and J. C. Willems, Eds. Birkhauser, 1993.

[4] L. Cuiyan, Z. Dongchun, and Z. Xianyi, "A survey of repetitive control," in *Proc. of IEEE/RSJ Intl. Conf. on IROS*, 2004, pp. 1160–1166.

[5] B. A. Francis and W. M. Wonham, "The internal model principle for linear multivariable regulators,," *J. Appl. Math. Optimiz.*, vol. 2, pp. 170–194, 1975.

[6] Y. Yamamoto and S. Hara, "Relationships between internal and external stability for infinite-dimensional systems with applications to servo problem," *IEEE Trans. Automat. Contr.*, vol. 33, no. 11, pp. 1044–1052, 1988.

[7] S. Hara, T. Omata, and M. Nakano, "Synthesis of repetitive control systems and its application," in *Proc. of 24th IEEE CDC*, 1985, pp. 1387–1392.

[8] K.-K. Chew and M. Tomizuka, "Digital control of repetitive errors in disk drive systems," *IEEE Control Syst. Mag.*, vol. 10, no. 1, pp. 16–20, 1990.

[9] S. Hara, M. Tetsuka, and R. Kondo, "Ripple attenuation in digital repetitive control systems," in *Proc. of 29th IEEE CDC*, 1990, pp. 1679–1684.

[10] H. Fujimoto, F. Kawakami, and S. Kondo, "Multirate repetitive control and applications," in *Proc. of ACC*, 2003, pp. 2875–2880.

[11] T. Chen and B. Francis, *Optimal Sampled-Data Control Systems*. Springer, 1995.

[12] Y. Yamamoto, "Digital control," in *Wiley Encyclopedia of Electrical and Electronics Engineering*, J. G. Webster, Ed. John Wiley & Sons, 1999.

[13] A. Langari and B. A. Francis, "Sampled-data repetitive control systems," in *Proc. of ACC*, 1998, pp. 3234–3235.

[14] A. Langari, *Sampled-Data Repetitive Control Systems*. PhD dissertation, 1997.

[15] H. Ishii and Y. Yamamoto, "Periodic compensation for sampled-data H^∞ repetitive control," in *Proc. of 37th IEEE CDC*, 1998, pp. 331–336.

[16] G. O. C. Hean, M. Nagahara, and Y. Yamamoto, "Repetitive control via sampled-data H^∞ control," in *Proc. of IFAC Workshop on Periodic Control Systems*, 2004, pp. 561–565.

[17] H. Fujioka, Y. Kobayashi, and H. Hayakawa, "A design of sampled-data H_∞ repetitive control," *Trans. of the ISCIE*, vol. 19, no. 11, pp. 413–417, 2006 (in Japanese).

[18] M. Morari and E. Zafiriou, *Robust Process Control*. Prentice Hall, 1989.

[19] P. P. Khargonekar and Y. Yamamoto, "Delayed signal reconstruction using sampled-data control," in *Proc. of 35th IEEE CDC*, 1996, pp. 1259–1263.

[20] H. Ishii, Y. Yamamoto, and B. A. Francis, "Sample-rate conversion via sampled-data H^∞ control," in *Proc. of 38th IEEE CDC*, 1999, pp. 3440–3445.

[21] M. Nagahara and Y. Yamamoto, "A new design for sample-rate converters," in *Proc. of 39th IEEE CDC*, 2000, pp. 4296–4301.

[22] K. Kashima, Y. Yamamoto, and M. Nagahara, "Optimal wavelet expansion via sampled-data control theory," *IEEE Signal Processing Lett.*, vol. 11, no. 2, pp. 79–82, 2004.

[23] M. Nagahara and Y. Yamamoto, "Optimal design of fractional delay filters," in *Proc. of 42th IEEE CDC*, 2003, pp. 6539–6544.

[24] H. Kakemizu, M. Nagahara, A. Kobayashi, and Y. Yamamoto, "Noise reduction of JPEG images by sampled-data H^∞ optimal ε filters," in *Proc. of SICE Annual Conf.*, 2005, pp. 1080–1085.

[25] M. Nagahara, T. Wada, and Y. Yamamoto, "Design of oversampling delta-sigma DA converters via H^∞ optimization," in *Proc. of IEEE ICASSP*, vol. III, 2006, pp. 612–615.

[26] Y. Yamamoto, "A new approach to signal processing via sampled-data control theory," *Australian J. Electrical and Electronics Engineering*, vol. 2, no. 2, pp. 141–149, 2005.

[27] M. Nagahara, M. Ogura, and Y. Yamamoto, "A novel approach to repetitive control via sampled-data H^∞ filters," in *Proc. of ASCC*, 2009 (submitted).

[28] C. M. Bishop, *Pattern Recognition and Machine Learning*. Springer, 2006.

[29] A. Neumaier, "Solving ill-conditioned and singular linear systems: A tutorial on regularization," *SIAM Review*, vol. 40, pp. 636–666, 1998.

[30] G. C. Goodwin, "Inverse problems with constraints," in *Proc. of IFAC 15th World Congress*, 2002, pp. 35–48.

[31] Y. Yamamoto, A. G. Madievski, and B. D. O. Anderson, "Approximation of frequency response for sampled-data control systems," *Automatica*, vol. 35, pp. 729–734, 1999.

[32] L. Mirkin and G. Tadmor, "Yet another H^∞ discretization," *IEEE Trans. Automat. Contr.*, vol. 48, no. 5, pp. 891–894, 2003.

[33] P. P. Vaidyanathan, *Multirate Systems and Filter Banks*. Prentice Hall, 1993.