A Novel Approach to Repetitive Control via Sampled-data H^∞ Filters

Masaaki Nagahara[1], Masaki Ogura[2], and Yutaka Yamamoto[3]

***Abstract*— A new paradigm of digital signal processing has recently been proposed in view of sampled-data control theory. This filter, called sampled-data H^∞ filter, approximates a linear phase characteristic in continuous-time domain, that is, a pure delay. By this property, we propose a new repetitive controller including the sampled-data H^∞ optimal filter. Moreover, to achieve the stability, we introduce the Internal Model Control (IMC). Our method solves the problem of stability, implementability, and inter sample ripples in digital repetitive control. A numerical example shows the effectiveness of our method.**

I. INTRODUCTION

Repetitive control is a control scheme to be designed for tracking periodic reference signals or for rejecting periodic disturbances [1]. This method was first introduced in the control of magnetic power supply for proton synchrotron [2]. Since then, a number of theoretical and industrial studies have been made on repetitive control, see surveys [3], [4] and references in there.

In view of the internal model principle [5], [6], repetitive control system must include the periodic signal generator

$$Q_{\mathrm{c}}(s) = \frac{\mathrm{e}^{-Ls}}{1-\mathrm{e}^{-Ls}}$$

in the feedback loop. This enables us to track *arbitrary* periodic references of period L with zero steady-state error. There however arise two problems; stability and implementability.

The stability problem is due to the fact that the repetitive control system is a neutral delay-differential system. By this nature, the control system cannot be exponentially stable if the transfer function of the plant is strictly proper [1], [6]. To remedy this, modified repetitive control [7], [1] was proposed, in which the repetitive controller $Q_{\mathrm{c}}(s)$ is replaced by

$$Q_{\mathrm{c}}^{\mathrm{mod}}(s) = \frac{F_{\mathrm{c}}(s)\mathrm{e}^{-Ls}}{1-F_{\mathrm{c}}(s)\mathrm{e}^{-Ls}} \tag{1}$$

where $F_{\mathrm{c}}(s)$ is a lowpass filter with a cutoff frequency ω_{c}. This repetitive control enables us to exponentially stabilize strictly proper plants, and to well track reference signals up to the cutoff frequency ω_{c}.

The modified repetitive control is a good scheme for general plants, there however remains the second problem: implementability. This is due to the infinite dimensionality of the delay e^{-Ls}, which is difficult to implement in an analog device. To this problem, controller discretization is an effective solution. In this approach, the delay e^{-Ls} is conventionally replaced by a sampled-data system $\mathbf{H}_h z^{-m}\mathbf{S}_h$, where $\mathbf{H}_h$ and $\mathbf{S}_h$ are respectively the zero-order hold and the ideal sampler with sampling period h, and m is a positive integer satisfying $L = mh$. This leads to digital repetitive control [8] with repetitive controller

$$Q_{\mathrm{d}}(z) = \frac{z^{-m}}{1-z^{-m}}.$$

This is the signal generator for arbitrary *discrete-time* periodic signals of an integer period m, by which the control system can achieve perfect tracking[1] for the periodic signals on the sampling instants.

However, it has another problem: intersample ripples. A remedy for this is to adopt a generalized hold [9] or a multirate control [10]. On the other hand, the intersample behavior in digital repetitive control can be taken into account by modern *sampled-data control theory* [11], [12]. There have been several works on sampled-data repetitive control, see [13], [14], [15], [16], [17].

Motivated by these works, we propose a novel repetitive control which solves the three problems mentioned above;

- stability,
- implementability,
- and intersample ripples.

For the stability problem, we introduce the internal model control (IMC) [18], instead of the modified repetitive control. The principle of IMC is in incorporating the model in the controller to handle the intrinsic difficulty in the plant. In our case, the plant may be strictly proper and non-minimum phase (i.e., difficult to control), and hence we place the model into our repetitive controller.

The main idea of the present article is that by using the IMC structure, sampled-data H^∞ filters can be applied to effectively solve the three problem listed above simultaneously. This filter was first proposed in [19], and since then a number of works have appeared on various digital signal processing problems, such as sampling rate converters [20], [21], optimal wavelet expansion [22], fractional delay filters [23], JPEG noise reduction [24] and $\Delta\Sigma$ converters [25]. See also a survey paper [26].

The sampled-data H^∞ filter is a digital one, which is designed to approximate the time delay e^{-Ls} with respect to the H^∞ norm of the sampled-data error system. It is noted [26] that the conventional Shannon reconstruction causes

The authors are with Graduate School of Informatics, Kyoto University, Kyoto 606-8501, Japan

[1] nagahara@ieee.org

[2] ogura@acs.i.kyoto-u.ac.jp

[3] yy@i.kyoto-u.ac.jp

[1] Perfect tracking means that the steady-state tracking error is zero.

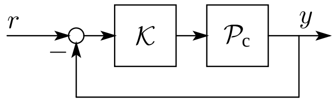

Fig. 1. Control System: $\mathcal{K}$ can be a continuous-time or a sampled-data repetitive controller, $\mathcal{P}_{\rm c}$ is a stable plant which can have unmodeled dynamics.

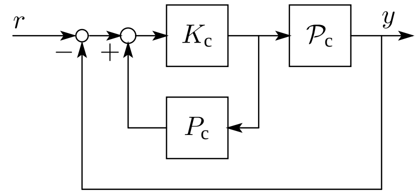

Fig. 2. Internal Model Control: $\mathcal{P}_{\rm c}$ is a stable continuous-time plant and $P_{\rm c}$ is a model of $\mathcal{P}_{\rm c}$. $K_{\rm c}$ is a continuous-time stable controller.

high-frequency noise due to the Gibbs phenomenon. This is because the conventional filter is designed to *perfectly* reconstruct the original signal within the Nyquist frequency. This is the same phenomenon as one in digital repetitive control. That is, the intersample ripples in digital repetitive control can be interpreted as the Gibbs phenomenon caused by perfectly tracking to the reference signal on the sampling instants. In view of this, the sampled-data H^∞ filter can also be very effective in digital repetitive control.

The organization of this article is as follows. In Section II, we formulate the problem of repetitive controller design. We adopt the IMC structure for repetitive control in continuous-time domain in Section III. Based on the discussion there, we introduce in Section IV the sampled-data H^∞ filter in repetitive control. Section V illustrates a design example to show effectiveness of our method. Section VI concludes this article.

A. Notations

$\mathbb{R}$	: the real numbers.
$\mathbb{R}_+$	: the positive real numbers.
$\mathbb{C}$	: the complex numbers.
$\mathbb{C}_+$	: the complex numbers s which satisfies $\mathrm{Re}[s] > 0$.
$\mathrm{j}\mathbb{R}$	: the imaginary numbers.
H^∞	: the bounded analytic functions in $\mathbb{C}_+$.
RH^∞	: the bounded rational functions in $\mathbb{C}_+$.
RL^∞	: the bounded rational functions on $\mathrm{j}\mathbb{R}$.
L^2	: the square integrable functions on $\mathbb{R}_+$.
$\mathcal{B}(L^2)$	: the linear bounded operators in L^2.
$\mathbf{S}_h$	: the ideal sampler with period h.
$\mathbf{H}_h$	: the zero-order hold with period h.

II. Repetitive Control Problem

We here formulate repetitive control problem. First of all, let us see the control system shown in Fig. 1. In this figure, $\mathcal{P}_{\rm c}$ is a plant to be controlled. Throughout this paper, we assume that the plant $\mathcal{P}_{\rm c}$ is stable, or already stabilized by another controller. The plant can be perturbed due to uncertainty in modeling. The controller $\mathcal{K}$ we design is a continuous-time or a sampled-data system. In both cases, the controller should be achieve

- (robust) stability,
- and tracking reference r of period L.

As mentioned above, if the relative degree of $\mathcal{P}_{\rm c}$ is not zero, perfect tracking and stability never go together. Thus, we relax the strict requirement of tracking. Our problem is formulated as follows.

Problem 1: Find a stabilizing controller $\mathcal{K}$ such that the sensitivity function $\mathcal{S} = (I + \mathcal{P}_{\rm c}\mathcal{K})^{-1}$ is sufficiently small at frequencies

$$\omega_n := \frac{2\pi n}{L}, \quad n = 0, 1, 2, \dots, N$$

where N is a given positive integer.

This is the main idea of modified repetitive control. We however do not adopt the structure of (1), but introduce the internal model control.

III. Repetitive Control by Internal Model Control

A. Sensitivity optimization

Internal model control (IMC) is a powerful method to control a plant which is difficult to control due to delays, nonlinearities, etc. The IMC structure is shown in Fig. 2. In this figure, $\mathcal{P}_{\rm c}$ is a continuous-time plant and $P_{\rm c} \in RH^\infty$ is a model of $\mathcal{P}_{\rm c}$.

At first, assume that $P_{\rm c} = \mathcal{P}_{\rm c}$ (i.e., there is no model error). Then, by the theory of Youla parameterization [27], [28], the feedback system is internally stable if and only if the controller $K_{\rm c}$ is in H^∞. If $K_{\rm c} \in H^\infty$, the sensitivity function is given by

$$S_{\rm c}(s) = 1 - P_{\rm c}(s)K_{\rm c}(s).$$

To solve our problem (Problem 1), we employ a weighting function $W_{\rm c} \in RH^\infty$ which is a lowpass filter[2] with cutoff frequency $\omega_c > \omega_N = 2\pi N L^{-1}$ for given positive integer N. By this weighting function, we optimize the following objective function.

$$J_{\rm c}(K_{\rm c}) = \|(\mathrm{e}^{-Ls} - P_{\rm c}K_{\rm c})W_{\rm c}\|_\infty, \quad K_{\rm c} \in H^\infty. \qquad (2)$$

Roughly speaking, if this is sufficiently small, then we have

$$P_{\rm c}(\mathrm{j}\omega_n)K_{\rm c}(\mathrm{j}\omega_n) \approx \mathrm{e}^{\mathrm{j}\omega_n L} = 1, \quad n = 1, 2, \dots, N$$

and hence the sensitivity function $S_{\rm c}(s)$ will be sufficiently small at $\{\mathrm{j}\omega_n\}_{n=1,2,\dots,N}$.

The optimal controller $K_{\rm c}$ minimizes the H^∞ norm of the error system shown in Fig. 3. This is a standard one-block H^∞ control problem [27], [28]. In fact, we have the following lemma.

[2]We choose a lowpass $W_{\rm c}$ because we want to better track signals at low frequencies $\omega_1, \dots, \omega_N$.

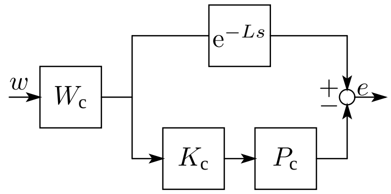

Fig. 3. Error system for approximation of e^{-Ls}.

Lemma 1: Assume P_c has no zeros on $\mathrm{j}\mathbb{R}$ and let

$$\gamma_\text{opt} := \inf_{K_\mathrm{c} \in H^\infty} J_\mathrm{c}(K_\mathrm{c}).$$

Then for any $\varepsilon > 0$, there exists $K_\varepsilon \in H^\infty$ such that

$$J_\mathrm{c}(K_\varepsilon) < \gamma_\text{opt} + \varepsilon.$$

Moreover, the sub optimal filter K_ε is decomposed as

$$K_\varepsilon = K_1 + K_2$$

where $K_1 \in RH^\infty$ and $K_2 \in H^\infty$.
The proof is given in the appendix.

B. Robust stability conditions

We here consider uncertainty in the plant $\mathcal{P}_\mathrm{c}$. Let

$$\mathbf{\Delta}_\mathrm{c} := \{\Delta_\mathrm{c} \in H^\infty : \|\Delta_\mathrm{c}\|_\infty \leq 1\}.$$

Then we have the following lemma.

Lemma 2: Assume $K_\mathrm{c} \in H^\infty$.

1) Assume the plant has multiplicative perturbations, that is,

$$\mathcal{P}_\mathrm{c} = P_\mathrm{c}(1 + \Delta_\mathrm{c} W_\mathrm{m}), \quad \Delta_\mathrm{c} \in \mathbf{\Delta}_\mathrm{c}$$

where $W_\mathrm{m} \in H^\infty$ is a weighting function. Then the control system shown in Fig. 2 is internally stable for all $\Delta_\mathrm{c} \in \mathbf{\Delta}_\mathrm{c}$ if

$$\|P_\mathrm{c} K_\mathrm{c} W_\mathrm{m}\|_\infty < 1.$$

2) Assume the plant has additive perturbations, that is,

$$\mathcal{P}_\mathrm{c} = P_\mathrm{c} + \Delta_\mathrm{c} W_\mathrm{a}, \quad \Delta_\mathrm{c} \in \mathbf{\Delta}_\mathrm{c}$$

where $W_\mathrm{a} \in H^\infty$ is a weighting function. Then the control system shown in Fig. 2 is internally stable for all $\Delta_\mathrm{c} \in \mathbf{\Delta}_\mathrm{c}$ if

$$\|K_\mathrm{c} W_\mathrm{a}\|_\infty < 1.$$

The proof is straightforward by using the small gain theorem [27].

The controller given in this section is the (sub) optimal solution for the repetitive control with an IMC structure shown in Fig. 2. It however includes the infinite-dimensional $K_2 \in H^\infty$ and is difficult to implement. In the next section, we solve this problem by using sampled-data H^∞ filters.

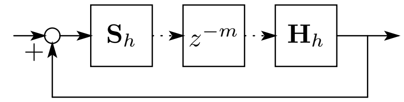

Fig. 4. Digital Repetitive Controller.

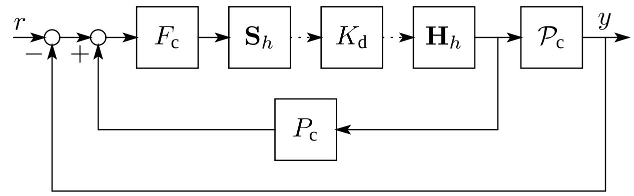

Fig. 5. Sampled-data repetitive control system with IMC structure.

IV. IMC Repetitive Control by sampled-data H^∞ filters

A. Sensitivity optimization

The basic digital repetitive controller is shown in Fig. 4. This is an approximation of the continuous-repetitive controller $Q_\mathrm{c}(s) = \mathrm{e}^{-mhs}(1 - \mathrm{e}^{-mhs})^{-1}$ assuming that m is a positive integer satisfying $L = mh$. By the equation $\mathbf{S}_h \mathbf{H}_h = I$ (identity) [11], the digital repetitive controller is given by $\mathbf{H}_h Q_\mathrm{d} \mathbf{S}_h$ where $Q_\mathrm{d}(z) = z^{-m}(1 - z^{-m})^{-1}$. This Q_d is the signal generator for arbitrary discrete-time periodic signals of period m. By the discrete-time version of internal model principle, the repetitive control system with Q_d can perfectly track arbitrary m-periodic references. As mentioned in the introduction, this system can lead to intersample ripples. We here propose an alternative to Q_d via IMC and sampled-data H^∞ filters. The proposed repetitive control system is shown in Fig. 5 (cf. Fig. 2). In this figure, F_c is a strictly proper RH^∞ function. This F_c is usually called anti-aliasing filter. This however is not necessarily required to have cutoff frequency within the Nyquist frequency πh^{-1}. This is introduced to make the feedback system be L^2-bounded. See [11]. The sampled-data H^∞ optimal filter is an approximation of the continuous-time delay e^{-Ls} taking account of intersample errors by using sampled-data H^∞ control theory. This fact can be used in our IMC repetitive control.

Assume that $P_\mathrm{c} = \mathcal{P}_\mathrm{c}$, and the sampling frequency $2\pi h^{-1}$ is non-pathological [11]. Then, by using the Youla parameterization, the feedback system is stable if and only if the digital controller K_d is stable. If K_d is stable, the sensitivity function is given by

$$S_\text{sd} = I - P_\mathrm{c} \mathbf{H}_h K_\mathrm{d} \mathbf{S}_h F_\mathrm{c}.$$

To solve our problem, we use the same weighting function W_c as one in the previous section. By this weighting function, we optimize the following objective function.

$$J_\text{sd}(K_\mathrm{d}) = \|(\mathrm{e}^{-mhs} - P_\mathrm{c} \mathbf{H}_h K_\mathrm{d} \mathbf{S}_h F_\mathrm{c}) W_\mathrm{c}\|_\infty \qquad (3)$$

where the norm is the L^2-induced norm of the sampled-data error system which is equivalent to the H^∞ norm of

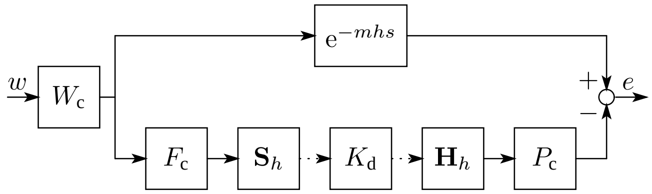

Fig. 6. Error system for approximation of e^{-mhs} where m is a positive integer.

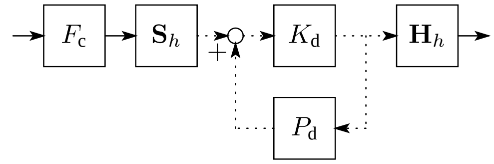

Fig. 7. Digital implementation of sampled-data repetitive controller: P_{d} is the step-invariant discretization of P_{c}.

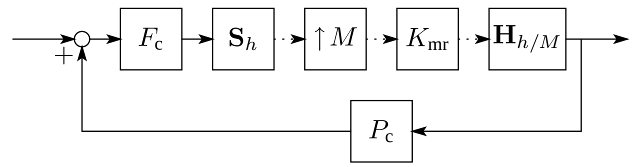

Fig. 8. Multirate repetitive controller with upsampler $\uparrow M$.

the lifted system [11]. Throughout this section, the norm $\|\cdot\|_\infty$ denotes the L^2-induced norm. The corresponding error system is shown in Fig. 6 (cf. Fig. 3). The optimal filter, called sampled-data H^∞ filter, can be obtained numerically via lifting method [29] or fast-discretization [19], [30].

B. Robust stability conditions

We here consider uncertainty in the plant $\mathcal{P}_{\mathrm{c}}$. Let

$$\boldsymbol{\Delta}_{\mathrm{c}} := \{\Delta_{\mathrm{c}} \in \mathcal{B}(L^2) : \|\Delta_{\mathrm{c}}\|_\infty \leq 1\},$$

where $\mathcal{B}(L^2)$ is the set of the linear bounded operators in L^2. We have the following lemma [11].

Lemma 3: Assume K_{d} is stable.

1) Assume the plant has multiplicative perturbations, that is,

$$\mathcal{P}_{\mathrm{c}} = P_{\mathrm{c}}(1 + \Delta_{\mathrm{c}} W_{\mathrm{m}}), \quad \Delta_{\mathrm{c}} \in \boldsymbol{\Delta}_{\mathrm{c}},$$

where $W_{\mathrm{m}} \in RH^\infty$ is a weighting function. Then the control system shown in Fig. 5 is internally stable for all $\Delta_{\mathrm{c}} \in \boldsymbol{\Delta}_{\mathrm{c}}$ if

$$\|P_{\mathrm{c}} \mathbf{H}_h K_{\mathrm{d}} \mathbf{S}_h F_{\mathrm{c}} W_{\mathrm{m}}\|_\infty < 1.$$

2) Assume the plant has additive perturbations, that is,

$$\mathcal{P}_{\mathrm{c}} = P_{\mathrm{c}} + \Delta_{\mathrm{c}} W_{\mathrm{a}}, \quad \Delta_{\mathrm{c}} \in \boldsymbol{\Delta}_{\mathrm{c}},$$

where $W_{\mathrm{a}} \in RH^\infty$ is a weighting function. Then the control system shown in Fig. 5 is internally stable for all $\Delta_{\mathrm{c}} \in \boldsymbol{\Delta}_{\mathrm{c}}$ if

$$\|\mathbf{H}_h K_{\mathrm{d}} \mathbf{S}_h F_{\mathrm{c}} W_{\mathrm{a}}\|_\infty < 1.$$

C. Implementation

Our sampled-data controller in Fig. 5 includes both a discrete-time controller K_{d} and a continuous-time plant model P_{c}. To implement this in a digital device, we *equivalently* transform the sampled-data system into a discrete-time system. To do this, we introduce the step-invariant transformation [11],

$$P_{\mathrm{d}} = \mathbf{S}_h F_{\mathrm{c}} P_{\mathrm{c}} \mathbf{H}_h.$$

This system is a linear time-invariant discrete-time system, by which our controller is transformed into

$$\mathcal{K} = \mathbf{H}_h \left(\frac{K_{\mathrm{d}}}{1 - P_{\mathrm{d}} K_{\mathrm{d}}} \right) \mathbf{S}_h F_{\mathrm{c}}$$

This controller is illustrated in Fig. 7.

D. Multirate control

It is pointed out in [10] that in digital repetitive control systems, the period of the hold device can shorter than that of the sampler. Considering this advantage, we can adopt a multirate controller shown in Fig. 8. instead of the controller in Fig. 5 In this system, $\uparrow M$ is the upsampler defined by

$$\uparrow M : \{x[k]\}_{k=0}^\infty \mapsto \{x[0], \underbrace{0, \ldots, 0}_{M-1}, x[1], 0, \ldots\}.$$

The controller K_{mr} is designed to optimize the following objective function

$$J_{\mathrm{mr}}(K_{\mathrm{mr}}) = \|\{e^{-mhs} - \mathbf{H}_{h/M} K_{\mathrm{mr}} (\uparrow M) \mathbf{S}_h F_{\mathrm{c}}\} W_{\mathrm{c}}\|_\infty \quad (4)$$

By using discrete-time lifting [11], this can be solved numerically, see [21]. Digital implementation as shown in Fig. 7 is also obtained by discrete-time lifting.

V. Design Example

In this section, we show a design example. The nominal plant P_{c} here is

$$P_{\mathrm{c}}(s) = \frac{s - 1}{(s+2)(s+3)(s+4)(s+5)}.$$

The period L of reference signals is $L = 10$. The sampling period is $h = 1$. The weighting function is a lowpass filter with cutoff frequency $\omega_c = \pi/20 = 0.157$, that is,

$$W_{\mathrm{c}}(s) = \frac{\pi}{20s + \pi}.$$

The lowpass filter F_{c} is chosen as

$$F_{\mathrm{c}}(s) = \frac{1}{Ts + 1}, \quad T = 1/100.$$

The parameter T_ε to design continuous-controller K_{c} (see the appendix) is the same as T. With these parameters, we design three controllers; the continuous-time H^∞ optimal K_{c} minimizing (2), the sampled-data H^∞ optimal K_{d} minimizing (3), and the multirate controller K_{mr} minimizing (4). Fig. 9 shows these controllers. In this figure, the inverse of P_{c} is also plotted. The continuous-time controller mimics

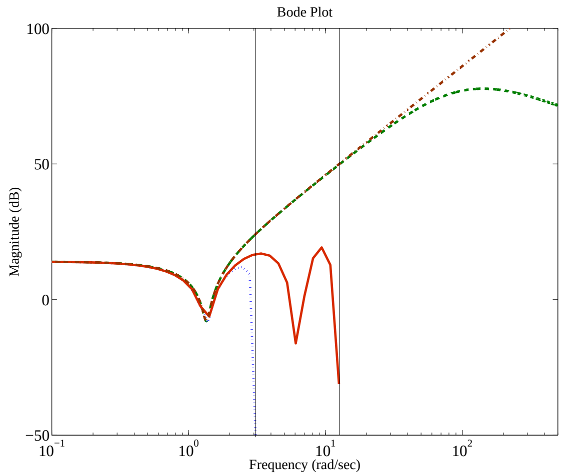

Fig. 9. Controllers; continuous-time H^∞ sub-optimal K_c (dash), the inverse of P_c (dash-dots), sampled-data H^∞ filter K_d (dots), and multirate sampled-data H^∞ filter K_mr (solid).

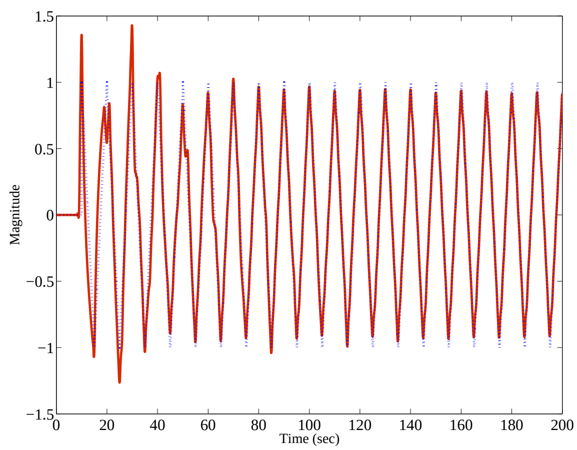

Fig. 10. Output of sampled-data H^∞ optimal repetitive control system; delayed reference (dash) and output y under plant perturbation.

the gain of P_c^{-1} very well. The two vertical lines show the Nyquist frequencies π (for K_d) and 4π (for K_mr). By using the sampled-data H^∞ optimal K_d, we simulate the repetitive control for a triangle input of period $L = 10$. We assume the plant has a multiplicative perturbation

$$\mathcal{P}_\mathrm{c}(s) = P_\mathrm{c}(s)\left(1 + \Delta_\mathrm{c}(s)\right), \quad \Delta_\mathrm{c}(s) = \frac{s-0.5}{s+1}.$$

Under this perturbation, the output is shown in Fig. 10. In the first few steps, the output shows over-shoots, then it converges the input (note that the input shown is delayed by one step). To see the error precisely, we show the error in Fig. 11. We also show the error when there is no perturbation. Since we did not consider the robust performance, the error with perturbation is much larger than that without perturbation.

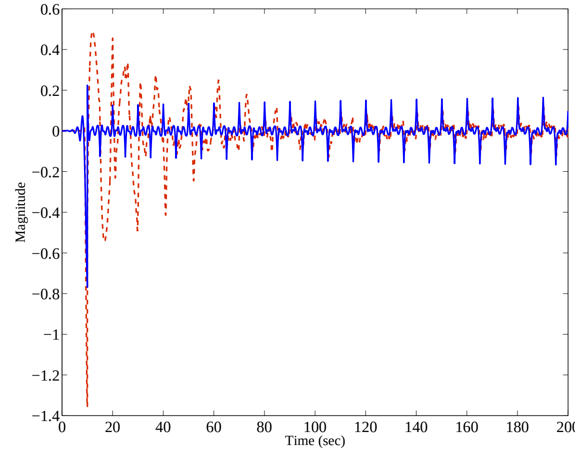

Fig. 11. Tracking Error; with no perturbation (solid) and perturbed (dash).

VI. Conclusion

In this article, we have proposed a novel repetitive control by and sampled-data H^∞ filters. The controller has the IMC structure and the controller optimizes the sensitivity function. A numerical example shows the effectiveness of our method. It is an easy extension to robust controller design.

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APPENDIX
PROOF OF LEMMA 1

Let (m, g) be an inner-outer factorization of $P_\mathrm{c}W_\mathrm{c} \in RH^\infty$, that is,

$$P_\mathrm{c}(s)W_\mathrm{c}(s) = m(s)g(s),$$

where m is inner and g is outer. Then by the method of absorption of the outer factor [31], the optimal value γ_opt is reduced to

$$\gamma_\mathrm{opt} = \inf_{Q \in H^\infty} \|\mathrm{e}^{-Ls} m^\sim W_\mathrm{c} - Q\|_\infty, \tag{5}$$

where $m^\sim(s) = m(-s)$. Let $\{A, B, C\}$ be a minimal realization of $m^\sim(s)W_\mathrm{c}(s)$. Then we have

$$\mathrm{e}^{-Ls} m^\sim(s) W_\mathrm{c}(s) = \Phi(s) + \Psi(s)$$

where

$$\begin{aligned}\Phi(s) &:= C(\mathrm{e}^{-Ls}I - \mathrm{e}^{-LA})(sI - A)^{-1}B,\\ \Psi(s) &:= C\mathrm{e}^{-LA}(sI - A)^{-1}B.\end{aligned}$$

Since Φ is an FIR (finite-impulse-response) system [32] (i.e., $\Phi \in H^\infty$) and Ψ is a rational function, we have

$$\gamma_\mathrm{opt} = \inf_{Q_1 \in RH^\infty} \|\Psi - Q_1\|_\infty. \tag{6}$$

By Nehari theorem [27], this value γ_opt is equal to the Hankel norm of Ψ, and let $Q_{1,\mathrm{opt}} \in RH^\infty$ be the optimal Q in (6) and let

$$Q_\mathrm{opt}(s) := Q_{1,\mathrm{opt}}(s) + \Phi(s).$$

This Q_opt is in H^∞ and is the optimal Q of (5). Let ν be the relative degree of the outer function $g(s)$. Since W_c is strictly proper and e^{-Ls} is inner, for any $\varepsilon > 0$, there exists $T_\varepsilon > 0$ such that

$$J_\mathrm{c}(K_\varepsilon) = \|\mathrm{e}^{-Ls}W_\mathrm{c} - mgK_\varepsilon\| < \gamma_\mathrm{opt} + \varepsilon,$$

where

$$\begin{aligned}K_\varepsilon(s) &:= \frac{Q_\mathrm{opt}(s)}{g(s)(T_\varepsilon s + 1)^\nu} = K_1(s) + K_2(s),\\ K_1(s) &= \frac{Q_{1,\mathrm{opt}}(s)}{g(s)(T_\varepsilon s + 1)^\nu} \in RH^\infty,\\ K_2(s) &= \frac{\Phi(s)}{g(s)(T_\varepsilon s + 1)^\nu} \in H^\infty.\end{aligned}$$

$\square$